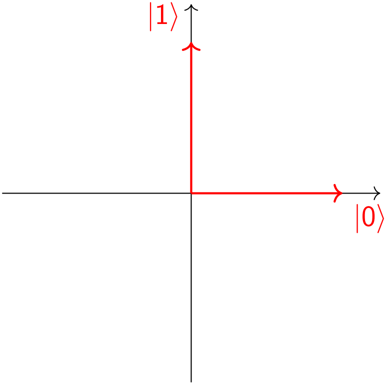
Completeness of the ZX-Calculus

Renaud Vilmart

Emmanuel Jeandel   Simon Perdrix

EJCIM
March 27th, 2018

- Any 2-dimensional quantum system. E.g. Photon polarisation:



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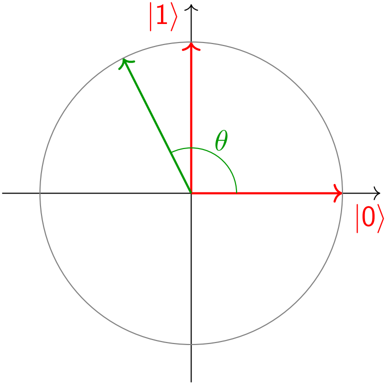

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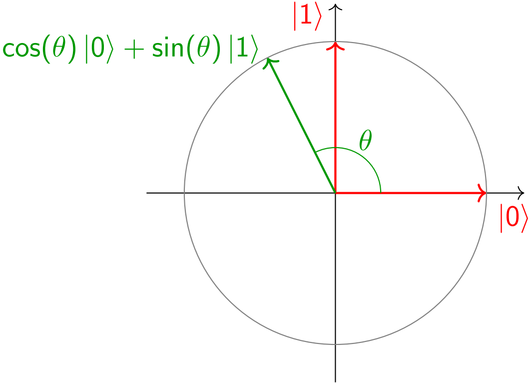

- Superposition of classical states: $\alpha\,|0\rangle + \beta\,|1\rangle$ where $\alpha, \beta \in \mathbb{C}$ and $|\alpha|^2 + |\beta|^2 = 1$.

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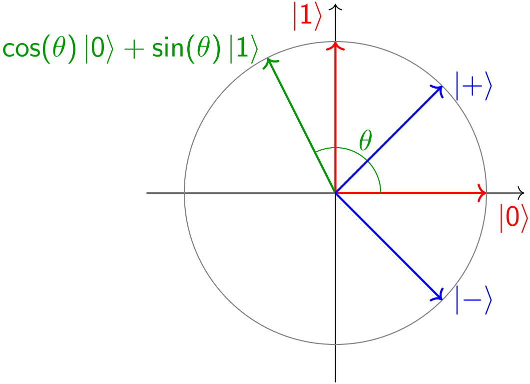

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- $\theta = \frac{\pi}{4}$: $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$ $\theta = -\frac{\pi}{4}$: $|-\rangle = \frac{|0\rangle-|1\rangle}{\sqrt{2}}$

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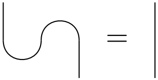
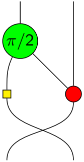
In ZX-Calculus: 

- Introduced by Coecke and Duncan in 2008



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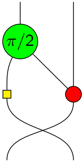

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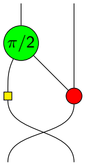

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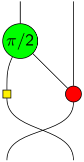

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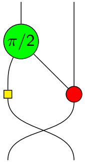

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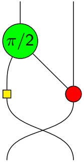

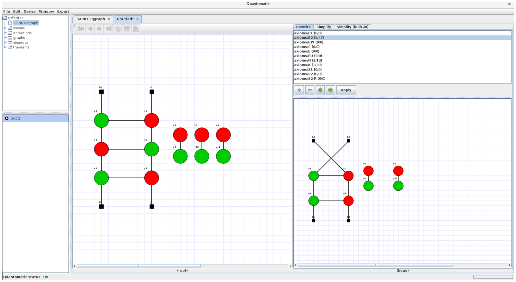

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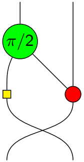
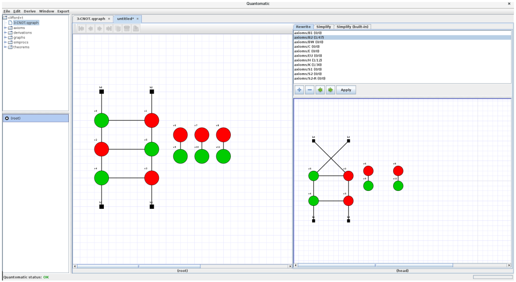
- Proof Assistant Quantomatic
- Study quantum error correction, lattice surgery, ...





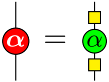
One-qubit operators:

Id: |

$R_z(\alpha)$:
rotation around Z

Hadamard:

$R_x(\alpha)$:
$= HR_z(\alpha)H$



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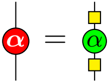
$= HR_z(\alpha)H$ 

States and projectors:

$\sqrt{2}\,|0\rangle$: 

$\sqrt{2}\,|1\rangle$: 

$|00\rangle + |11\rangle$: 

$\langle 00| + \langle 11|$: 

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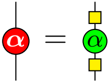
$= HR_z(\alpha)H$ 

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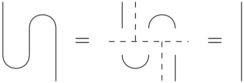
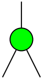
Topological equation:



Generalised green dot:

copy:  :

$$\begin{array}{ccc} |0\rangle & \mapsto & |00\rangle \\ |1\rangle & \mapsto & |11\rangle \end{array}$$

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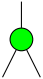
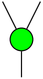
copy:  : $\begin{array}{ccc} |0\rangle & \mapsto & |00\rangle \\ |1\rangle & \mapsto & |11\rangle \end{array}$

co-copy: $= \text{copy}^\dagger$  : $\begin{array}{ccc} |00\rangle & \mapsto & |0\rangle \\ |01\rangle & \mapsto & 0 \\ |10\rangle & \mapsto & 0 \\ |11\rangle & \mapsto & |1\rangle \end{array}$

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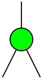
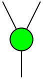
copy:  :

$$|0\rangle \mapsto |00\rangle$$
$$|1\rangle \mapsto |11\rangle$$

co-copy: $=$ copy†  :

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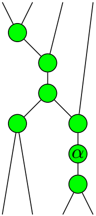

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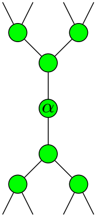

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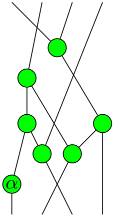
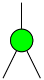


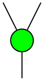
, $\cdots$:

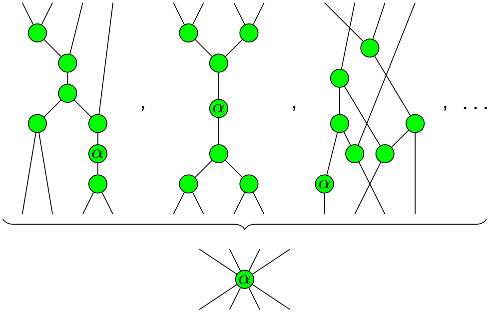
$$|0000\rangle \mapsto |0000\rangle$$
$$|1111\rangle \mapsto e^{i\alpha}\,|1111\rangle$$
$$\text{—} \mapsto 0$$

Intuition

Generalised green dot:

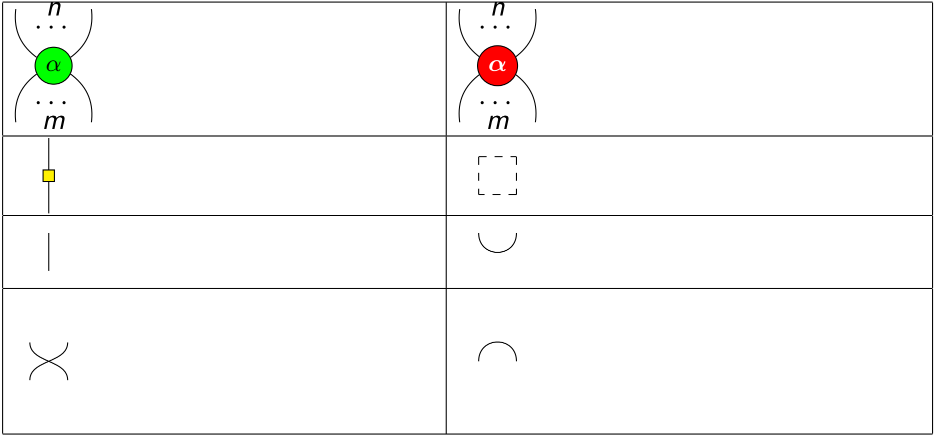
copy:  : $|0\rangle \mapsto |00\rangle$, $|1\rangle \mapsto |11\rangle$

co-copy: $= \text{copy}^\dagger$  : $|00\rangle \mapsto |0\rangle$, $|01\rangle \mapsto 0$, $|10\rangle \mapsto 0$, $|11\rangle \mapsto |1\rangle$



: $|0000\rangle \mapsto |0000\rangle$, $|1111\rangle \mapsto e^{i\alpha}|1111\rangle$, $\_\_ \mapsto 0$

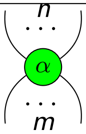
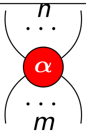
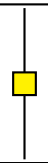
ZX-Calculus Generators and Semantics [Coecke,Duncan'08]



with $n, m \in \mathbb{N}$ and $\alpha \in \mathbb{R}$

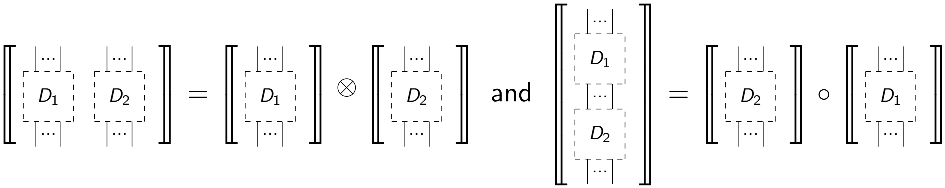
ZX-Calculus Generators and Semantics [Coecke,Duncan'08]

The standard interpretation $[\![.]\!]$:

	$\mapsto \begin{cases} \|0\cdots0\rangle \mapsto \|0\cdots0\rangle \\ \|1\cdots1\rangle \mapsto e^{i\alpha}\|1\cdots1\rangle \\ \_\_ \mapsto 0 \end{cases}$		$\mapsto \begin{cases} \|+\cdots+\rangle \mapsto \|+\cdots+\rangle \\ \|-\cdots-\rangle \mapsto e^{i\alpha}\|-\cdots-\rangle \\ \_\_ \mapsto 0 \end{cases}$
	$\mapsto \begin{cases} \|0\rangle \mapsto \|+\rangle \\ \|1\rangle \mapsto \|-\rangle \end{cases}$		$\mapsto 1$
	$\mapsto \begin{cases} \|0\rangle \mapsto \|0\rangle \\ \|1\rangle \mapsto \|1\rangle \end{cases}$		$\mapsto \langle 00\| + \langle 11\|$
	$\mapsto \begin{cases} \|00\rangle \mapsto \|00\rangle \\ \|01\rangle \mapsto \|10\rangle \\ \|10\rangle \mapsto \|01\rangle \\ \|11\rangle \mapsto \|11\rangle \end{cases}$		$\mapsto \|00\rangle + \|11\rangle$

with $n, m \in \mathbb{N}$ and $\alpha \in \mathbb{R}$

ZX-Calculus Compositions



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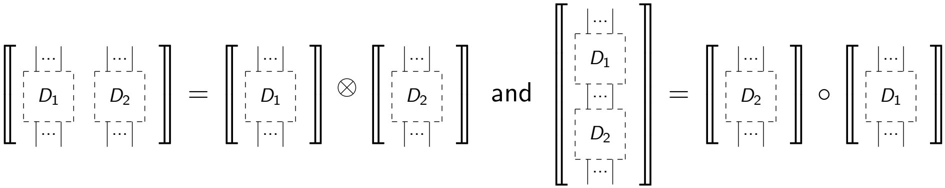

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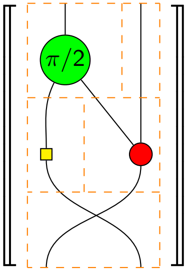

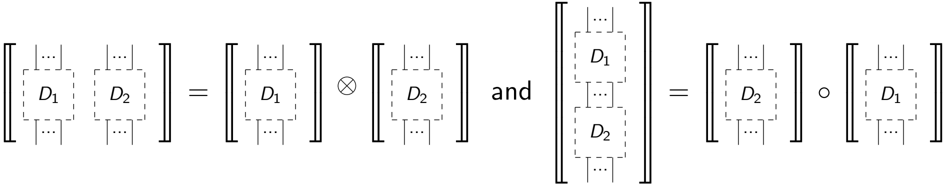

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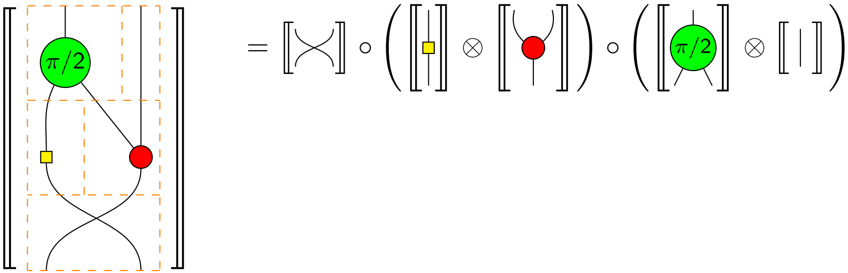

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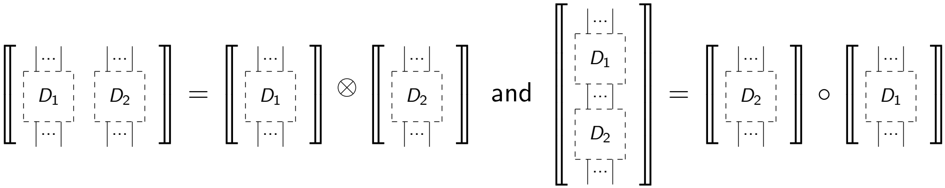

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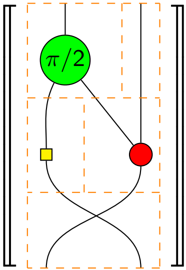
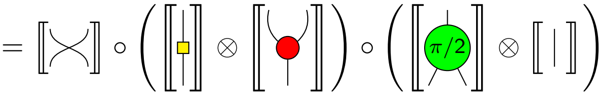
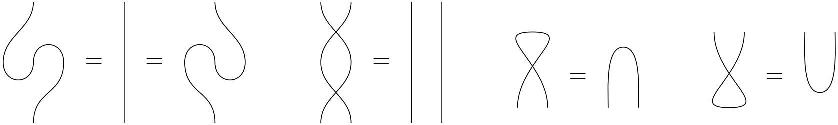
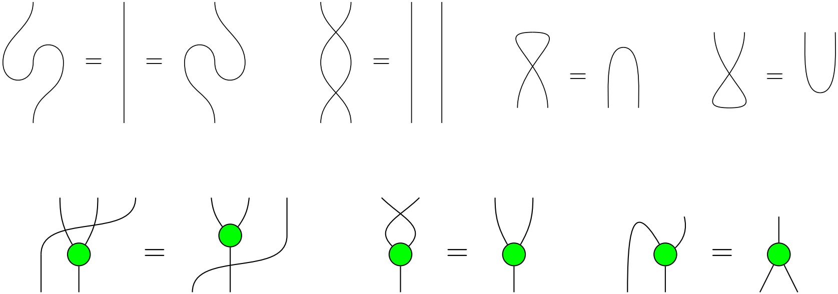


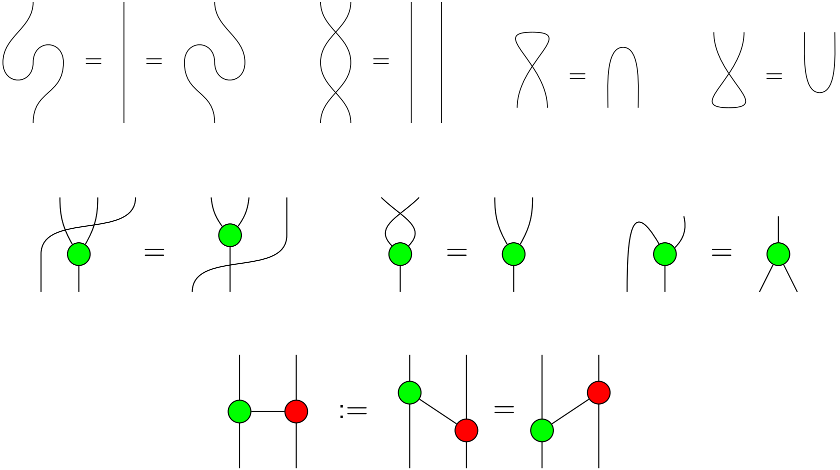


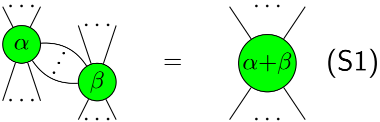
$$\propto \begin{cases} |00\rangle \mapsto |0+\rangle \\ |01\rangle \mapsto |1+\rangle \\ |10\rangle \mapsto i\,|1-\rangle \\ |11\rangle \mapsto i\,|0-\rangle \end{cases}$$

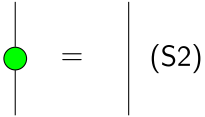




Only Topology Matters




α
β
=
α+β
(S1)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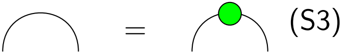
(S2)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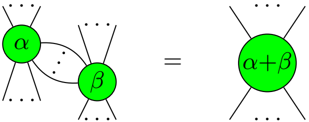
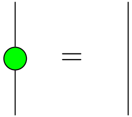
(S3)

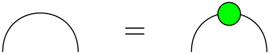
Spider Rules



(S1)

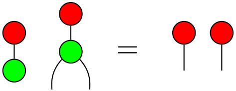


(S2)

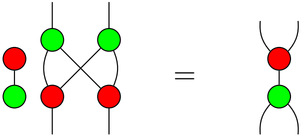


(S3)

Bialgebra Rules

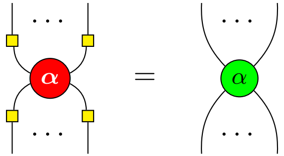


(B1)

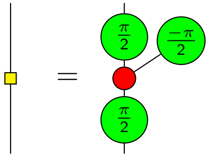


(B2)

Hadamard Rules

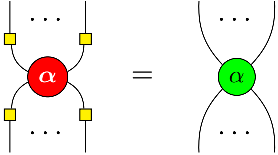


(H)



(EU)

Hadamard Rules



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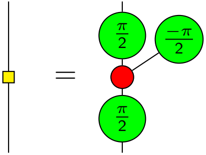

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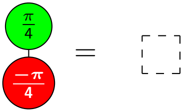
Additional Rules



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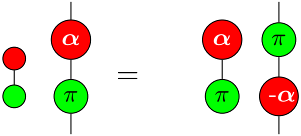

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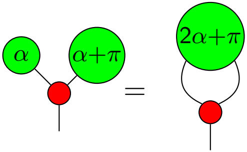

(SUP)

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$\frac{\pi}{p}$-fragment: the restriction to angles multiple of $\frac{\pi}{p}$

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Soundness, Completeness

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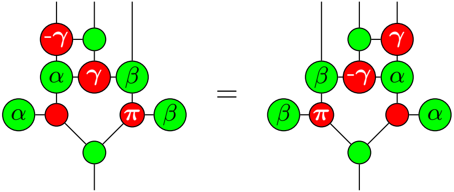
“$\frac{\pi}{4}$-Completeness” Rules



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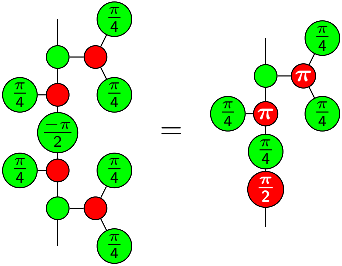

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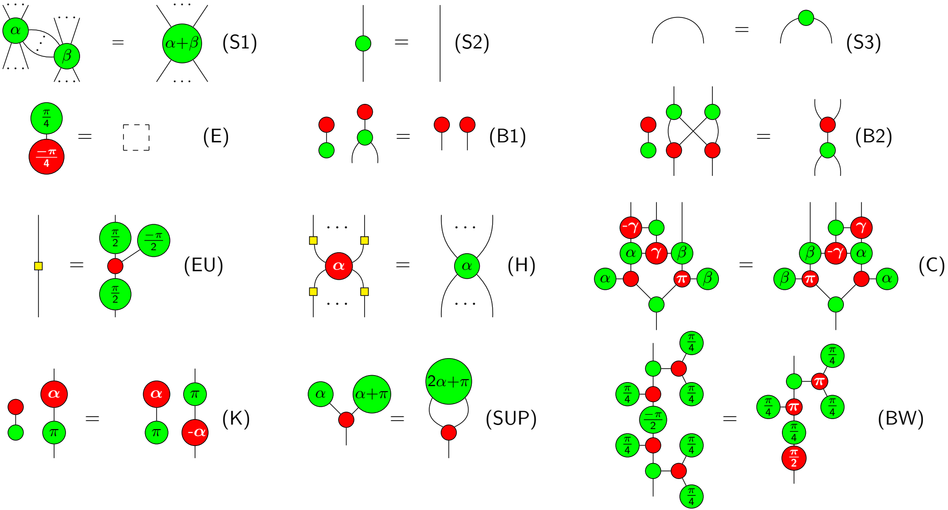
Set of Rules ZX$_{\pi/4}$ [Jeandel,Perdrix,Vilmart'17]



+ colour-swapped versions

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- First complete axiomatisation in general [Ng,Wang'17]: 2 additional generators, $\sim$20 additional axioms

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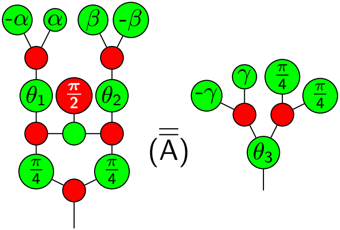
- Refinement [JPV'17b]: 0 new generator, 1 additional axiom:



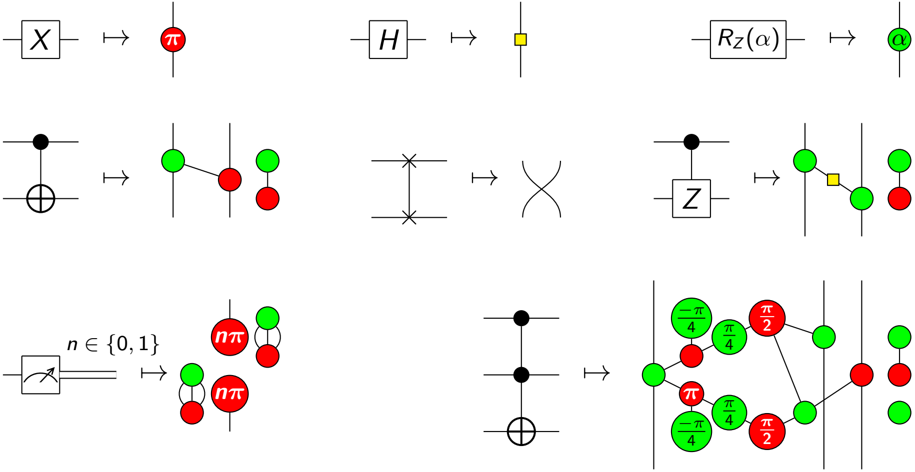
$$2e^{i\theta_3}\cos(\gamma) = e^{i\theta_1}\cos(\alpha) + e^{i\theta_2}\cos(\beta)$$

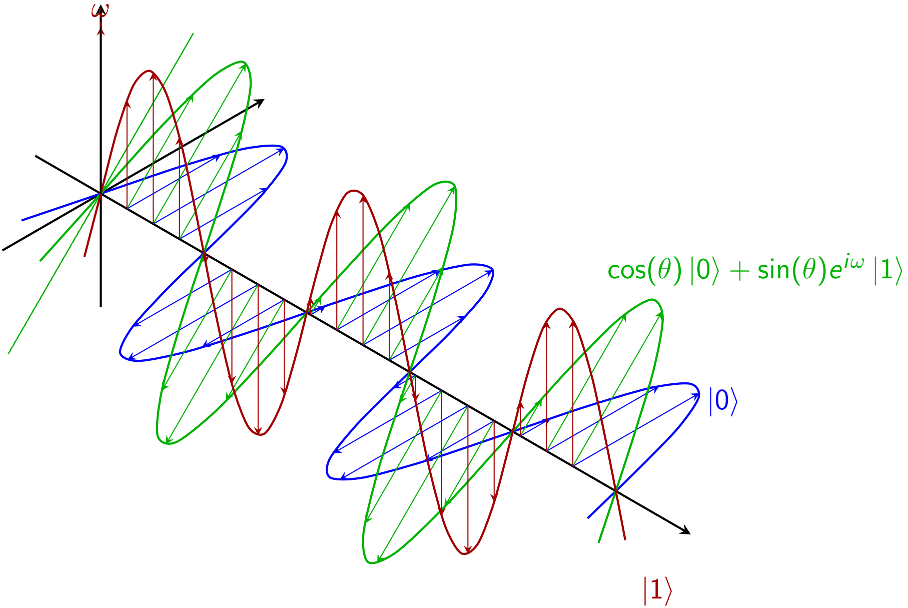
- Completeness for an approximately universal fragment
- Completeness in general

Remaining Questions:

- Necessity of the axioms
- (Pseudo-)normal form for ZX-diagrams
- Possibility to orient the axioms
- Completeness for other fragments
- Application to circuits

Circuits to ZX-diagrams





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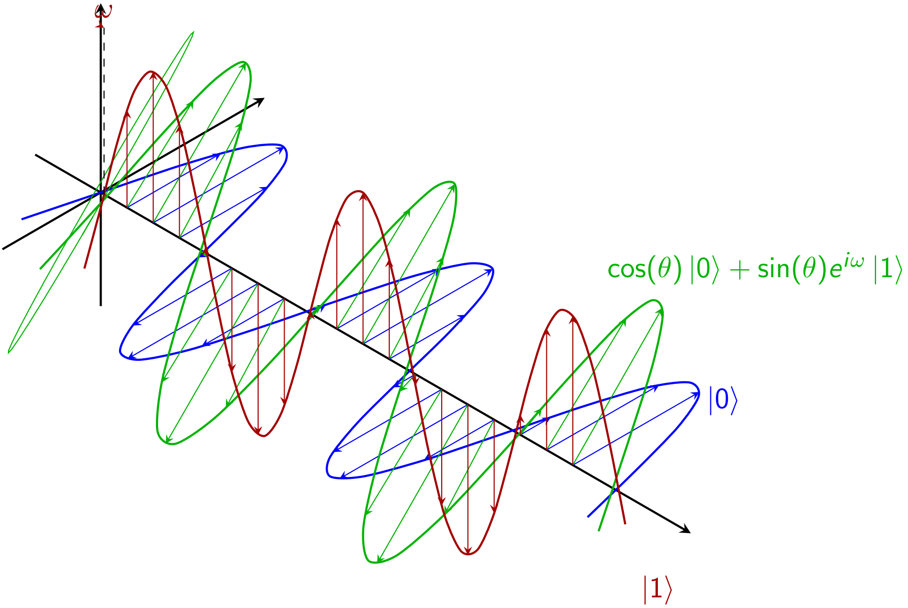
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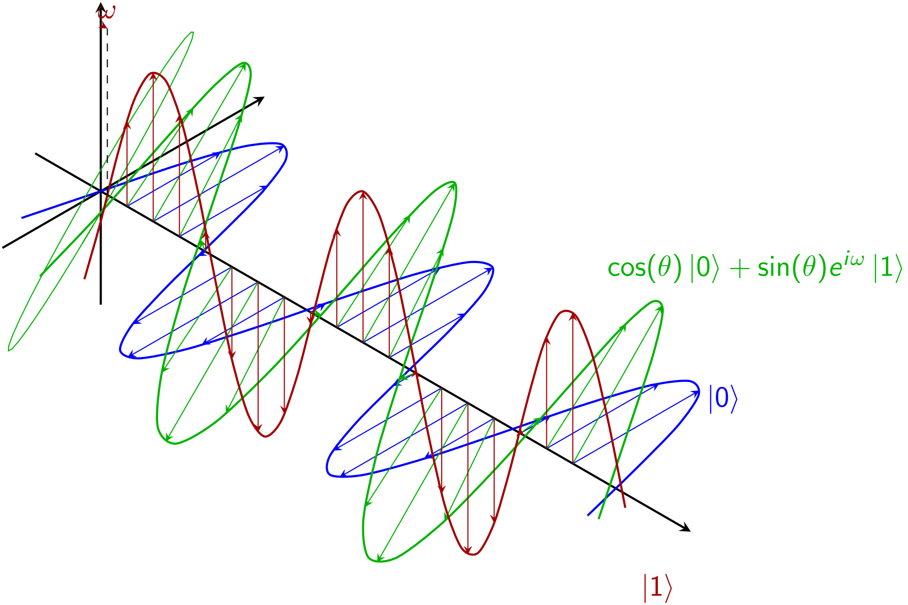
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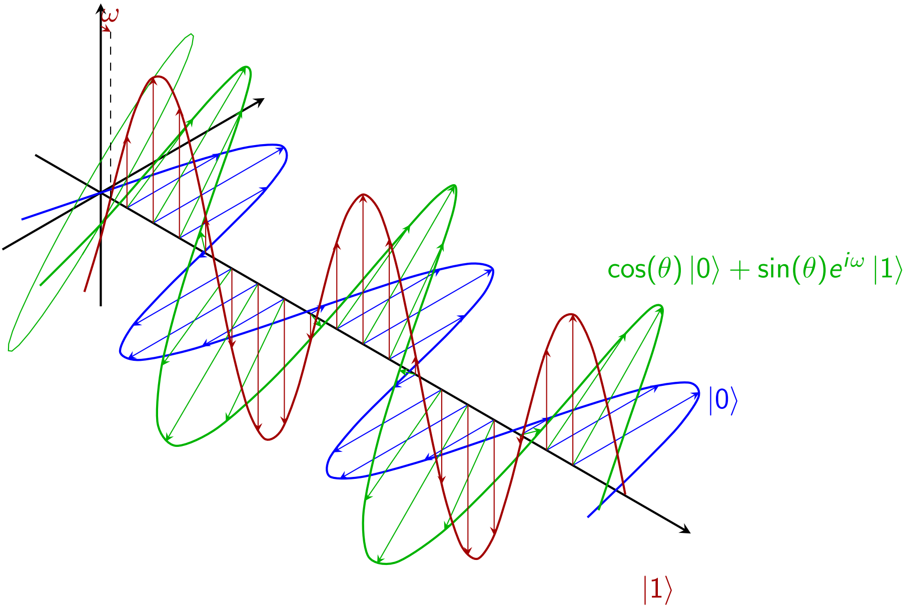
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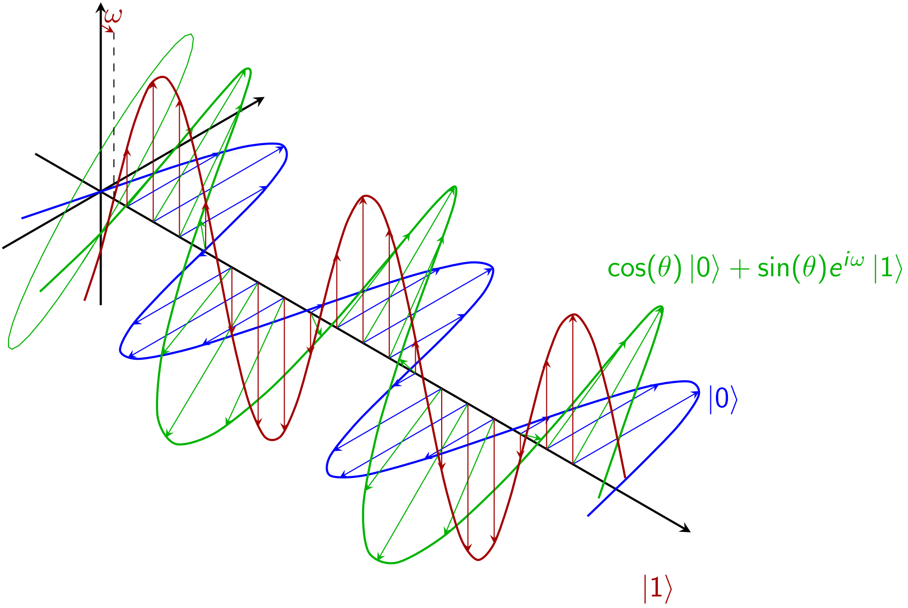

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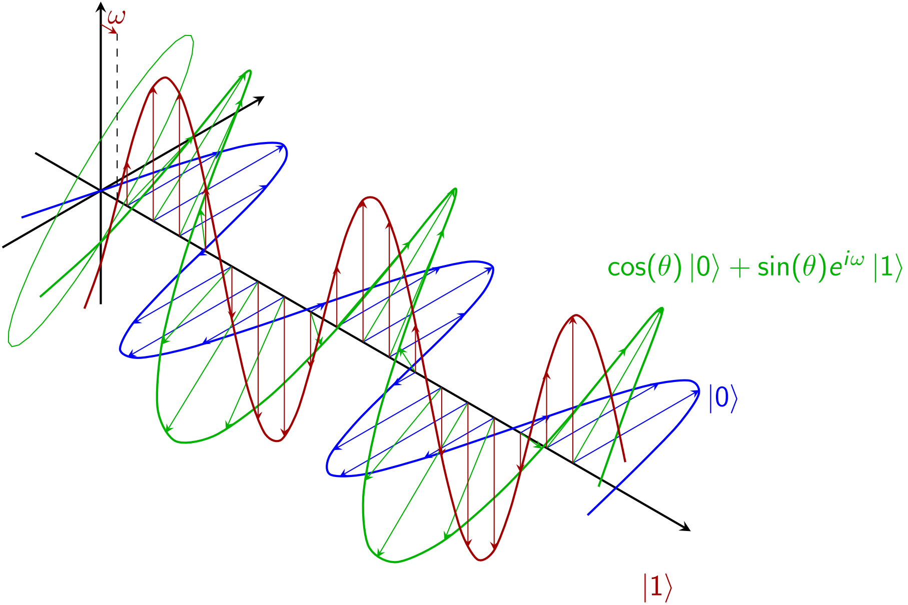
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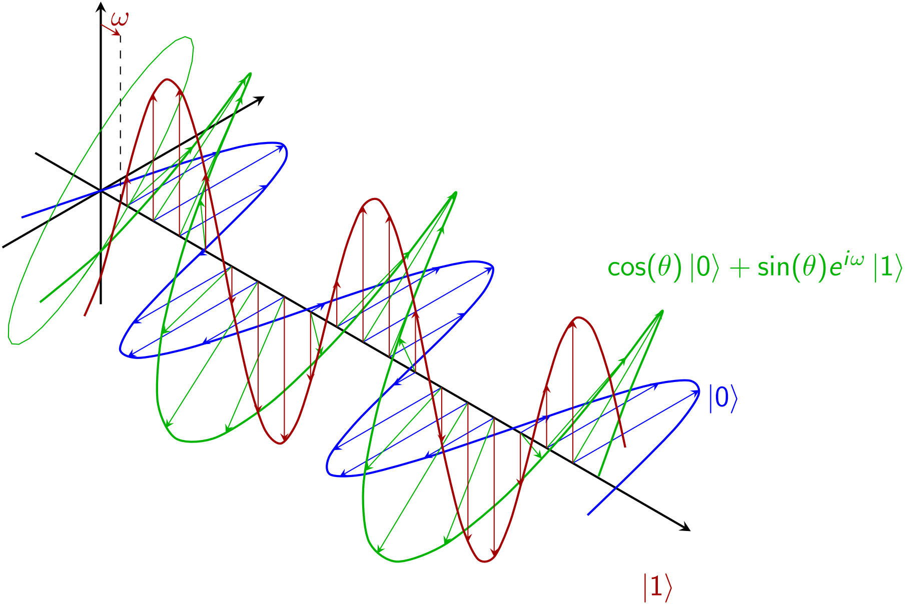

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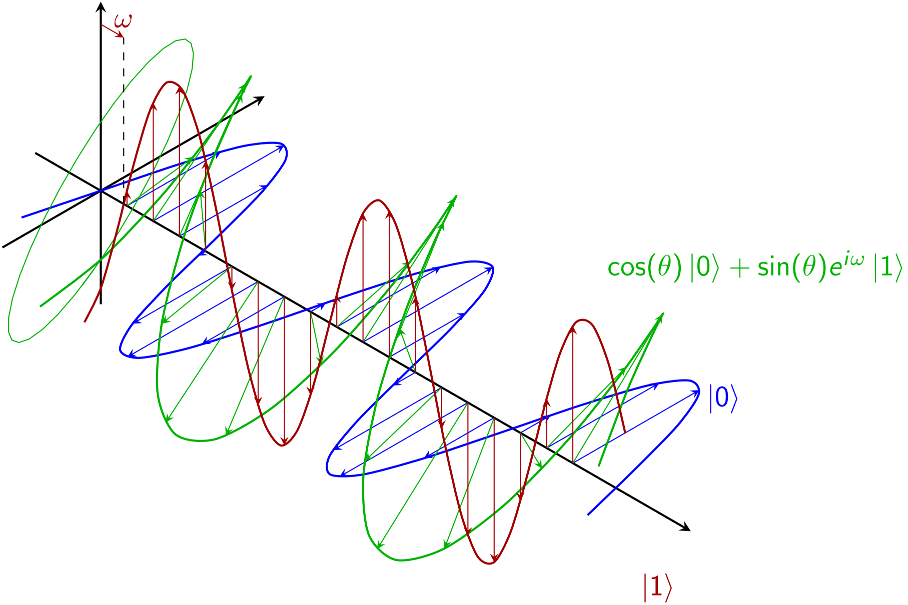

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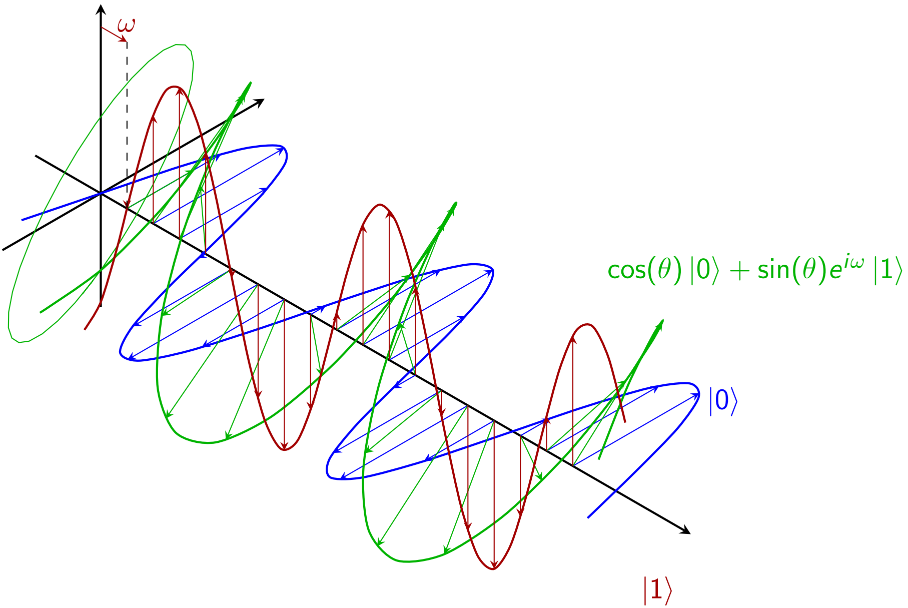
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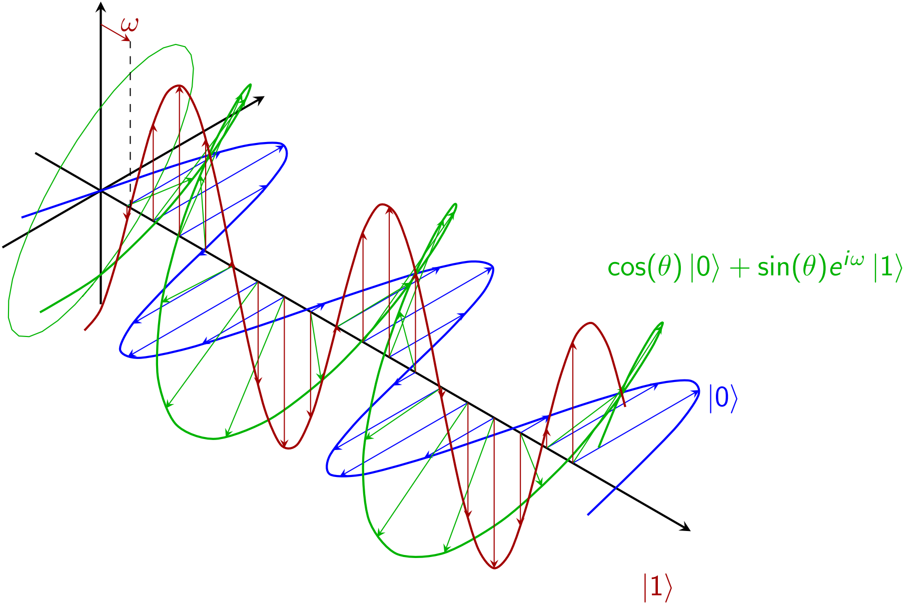

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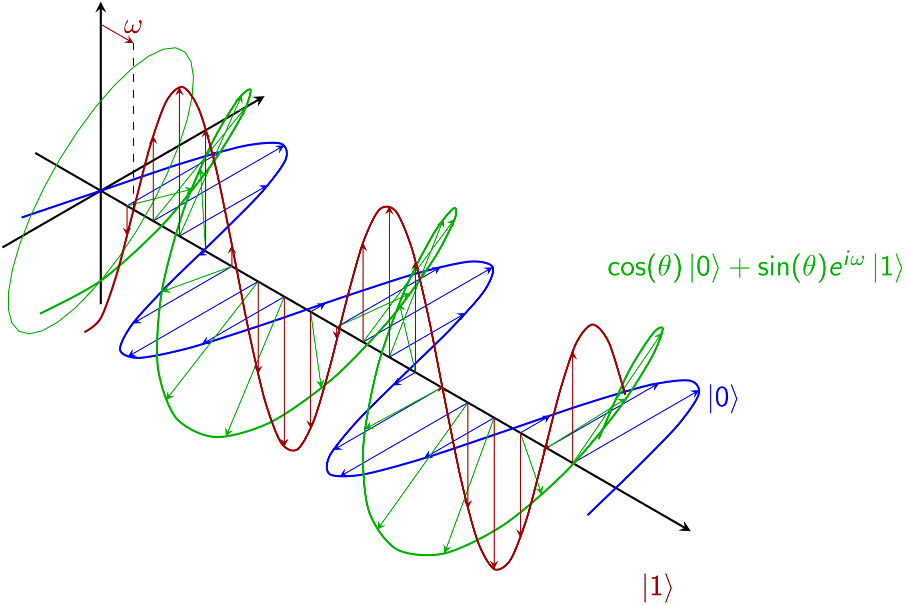

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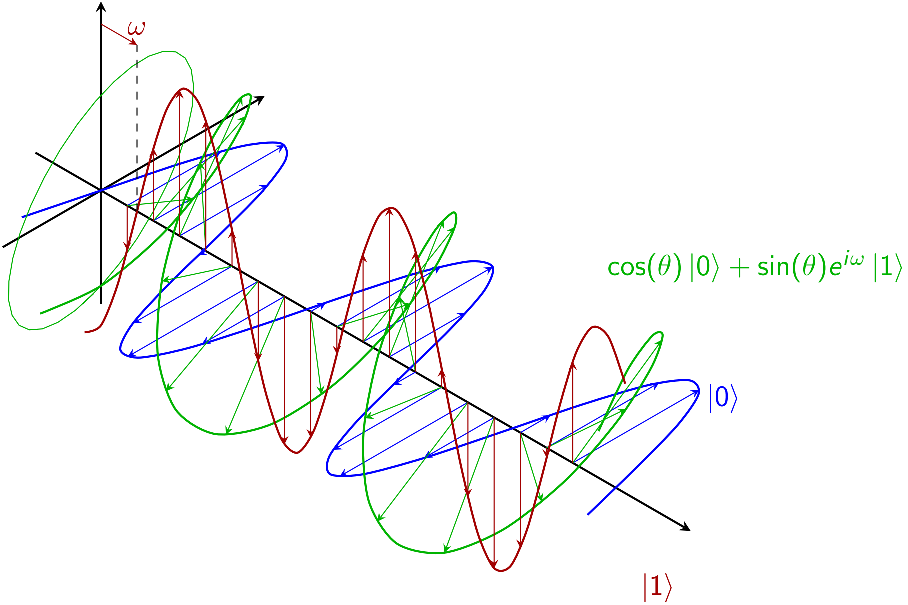

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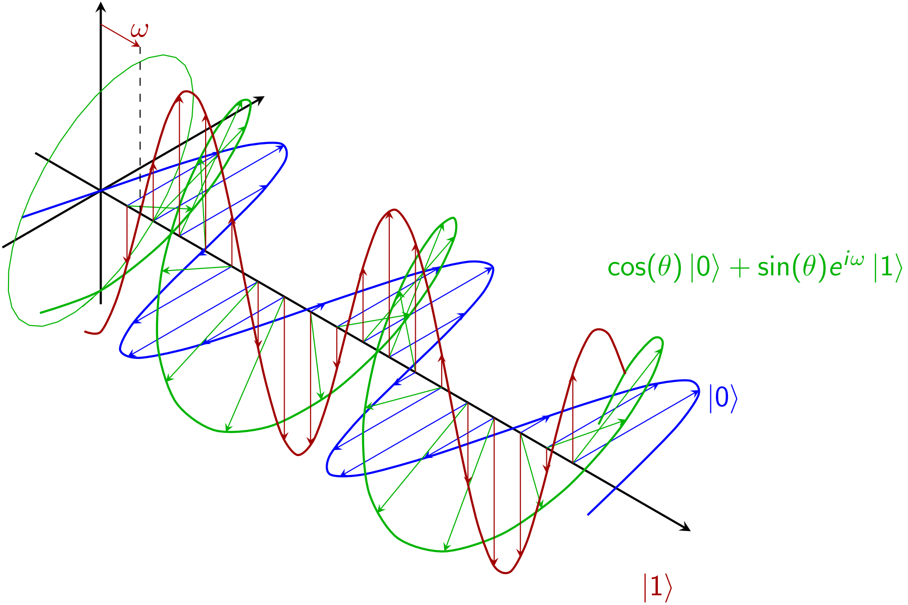
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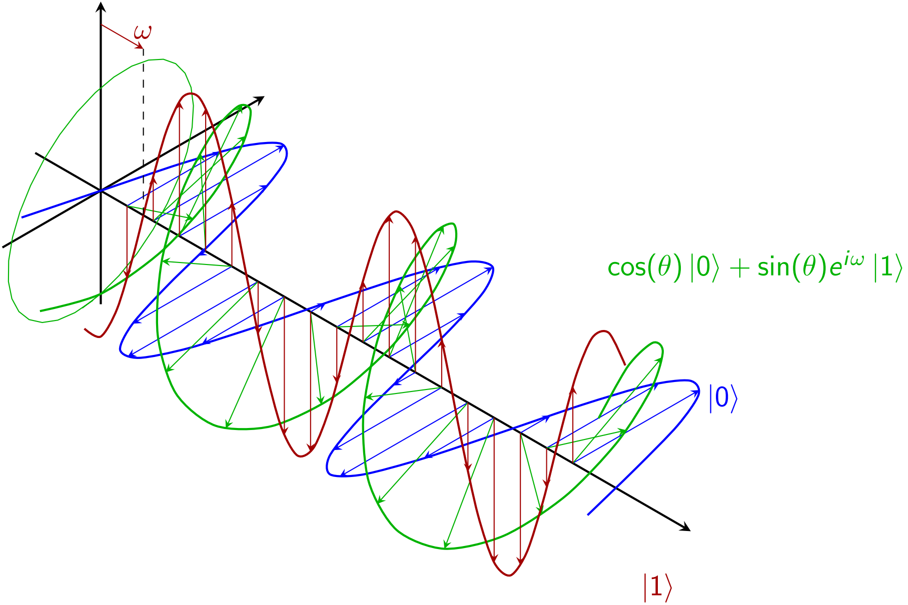

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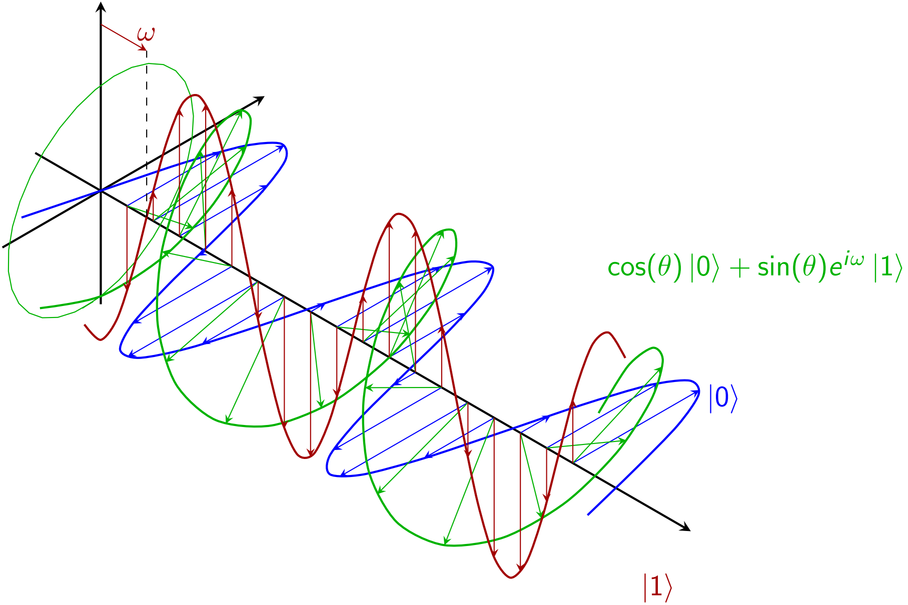
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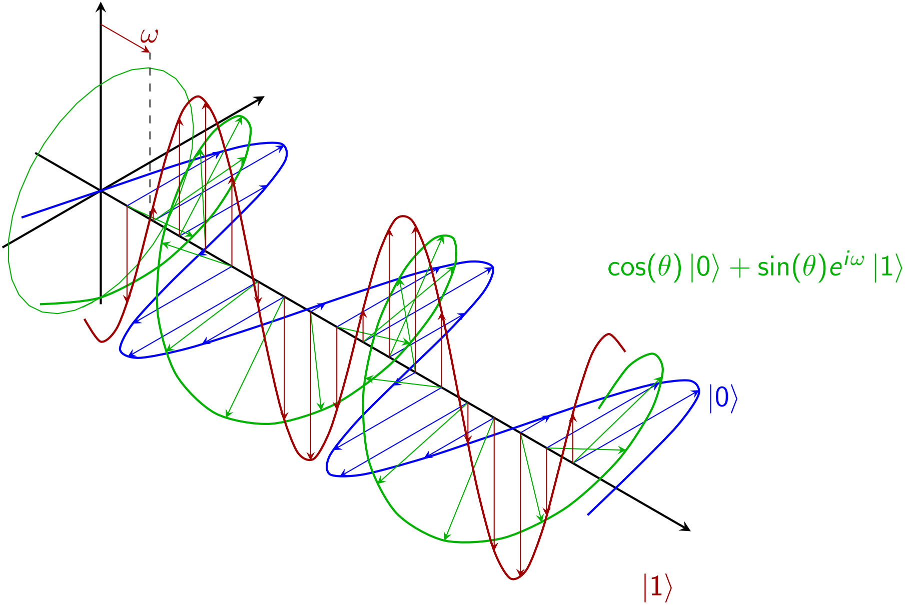

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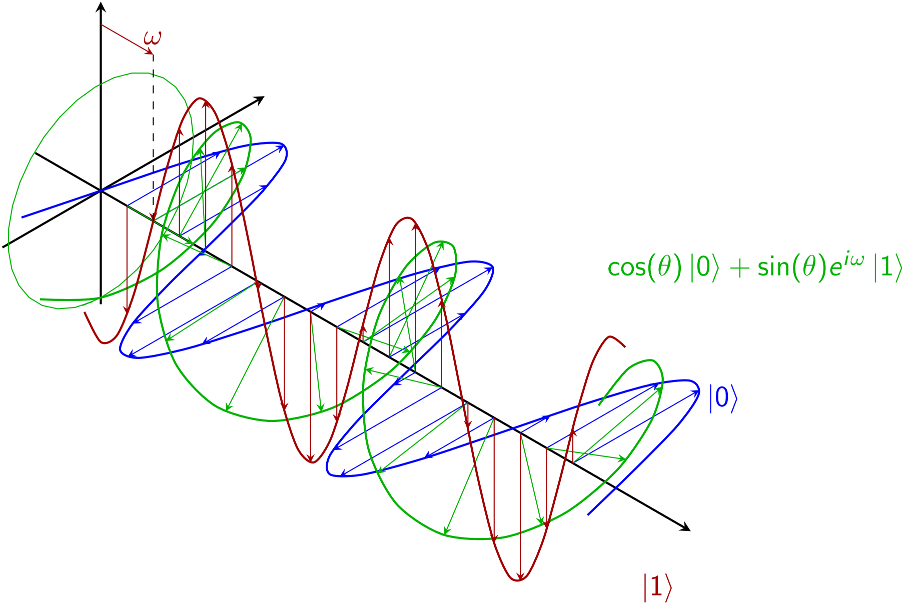
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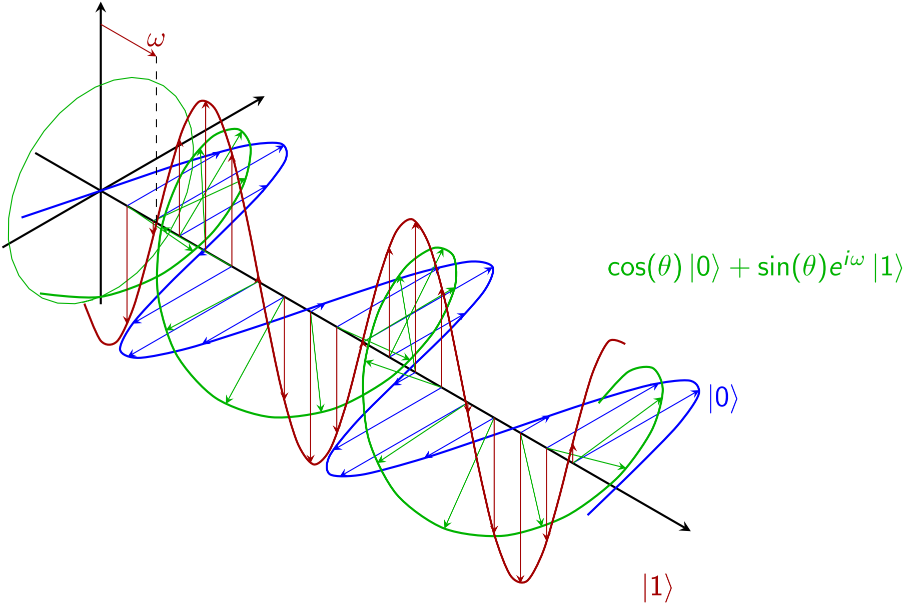
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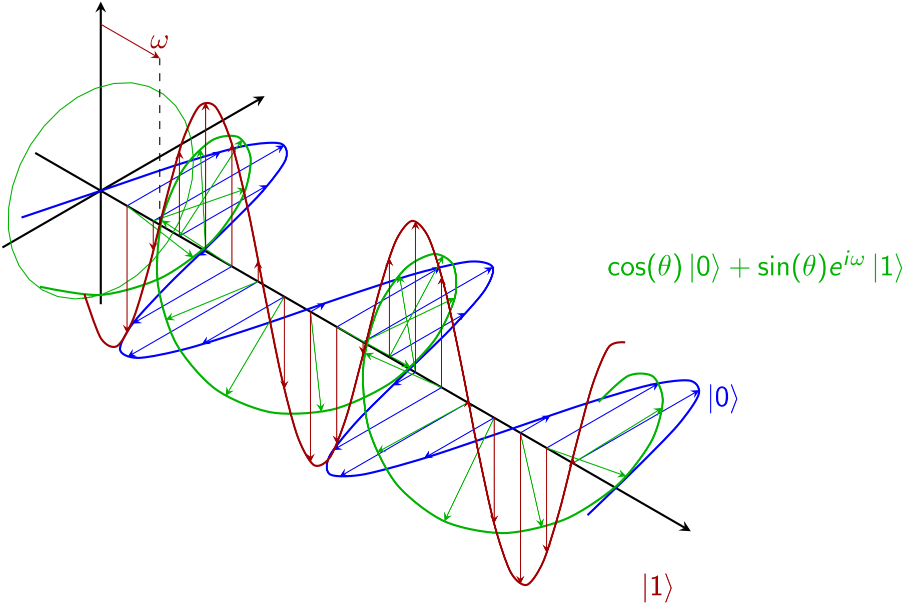
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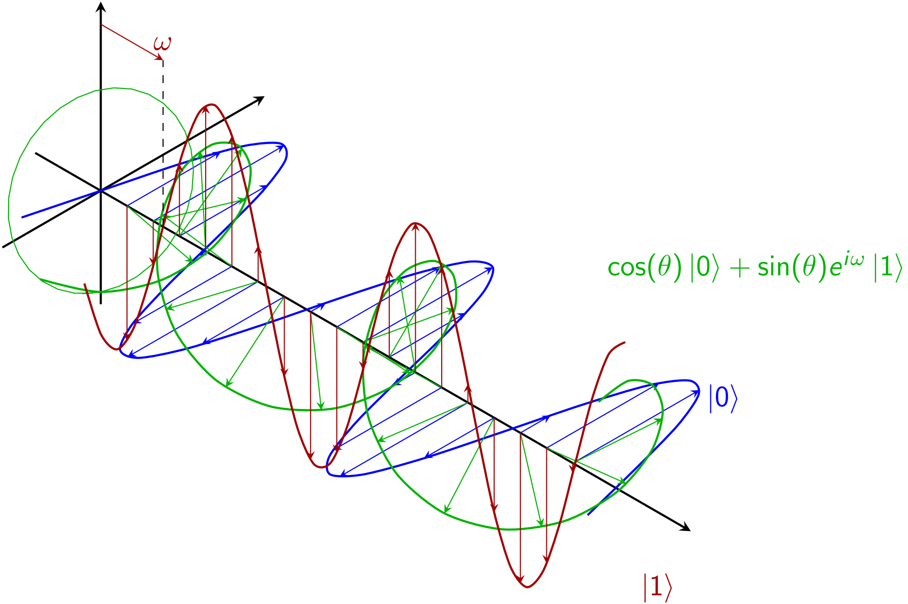
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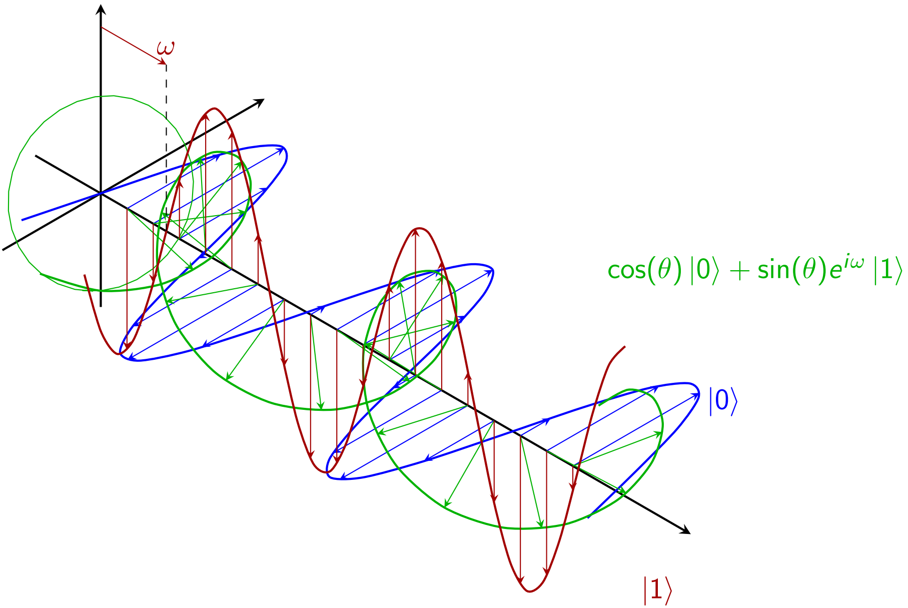

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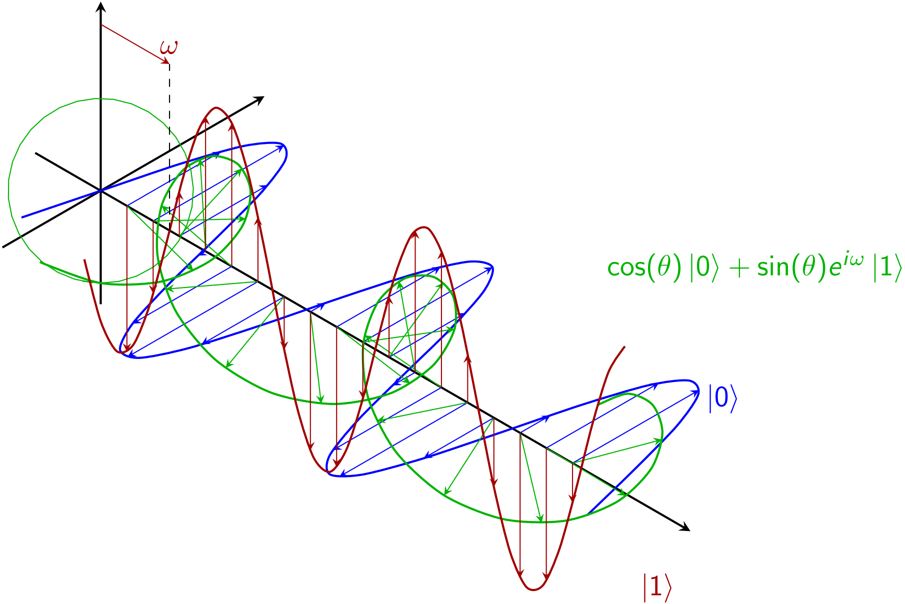
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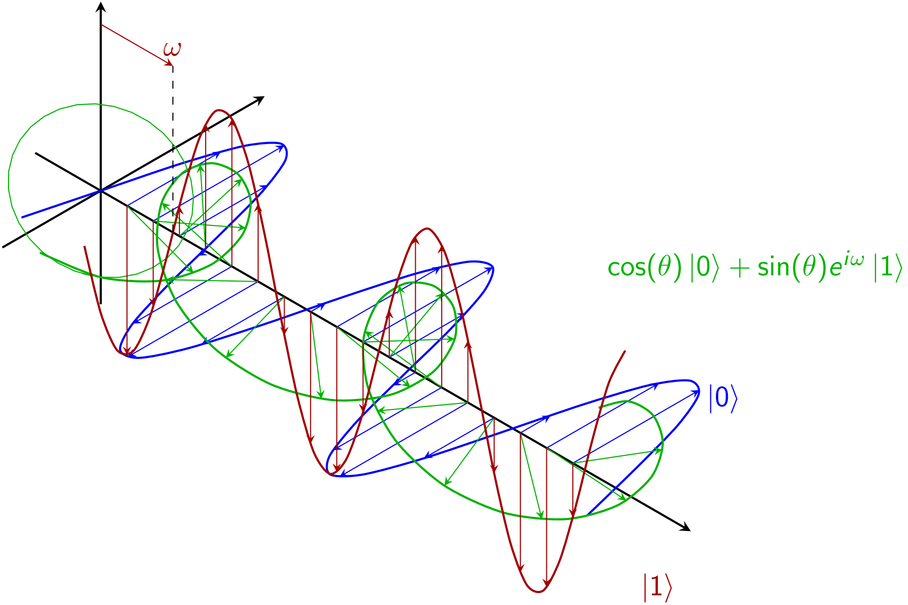
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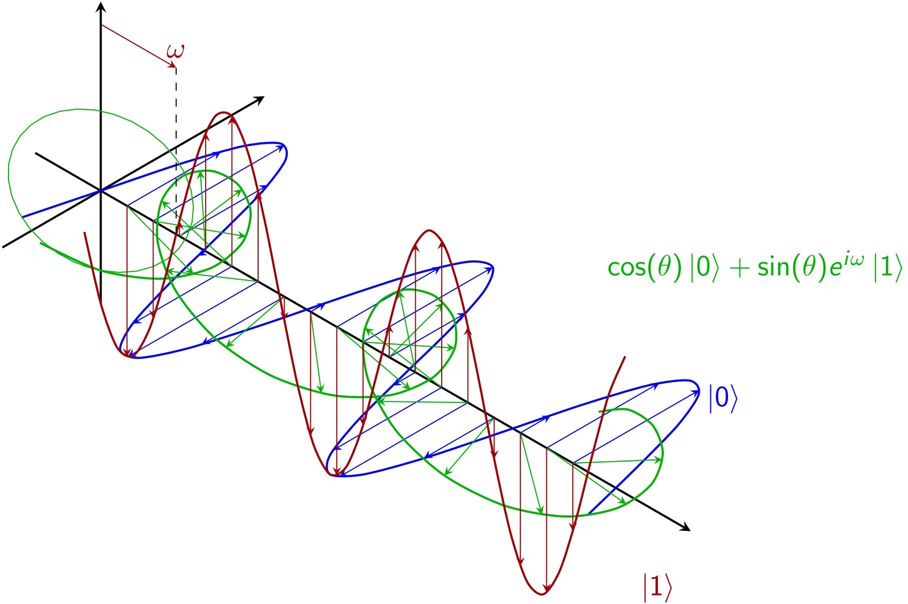
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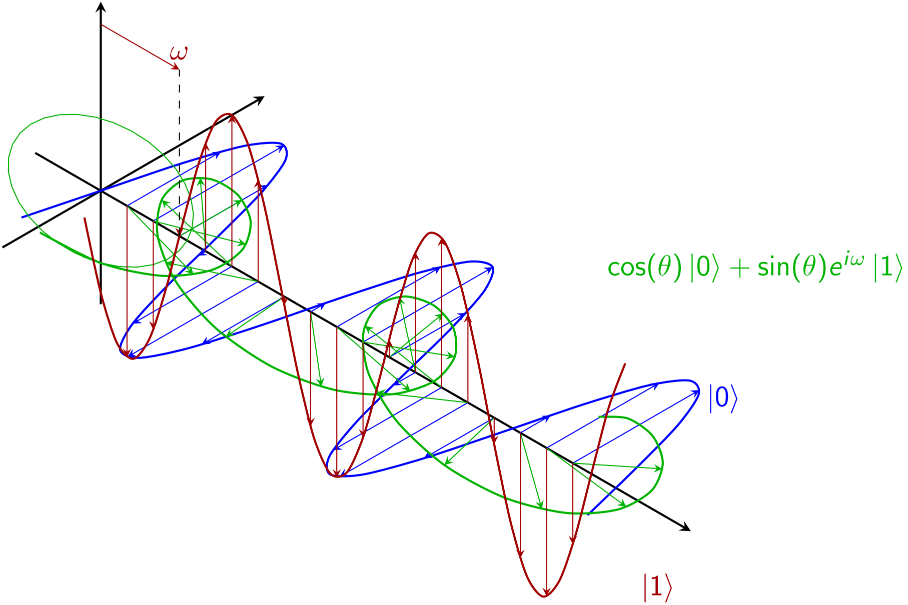
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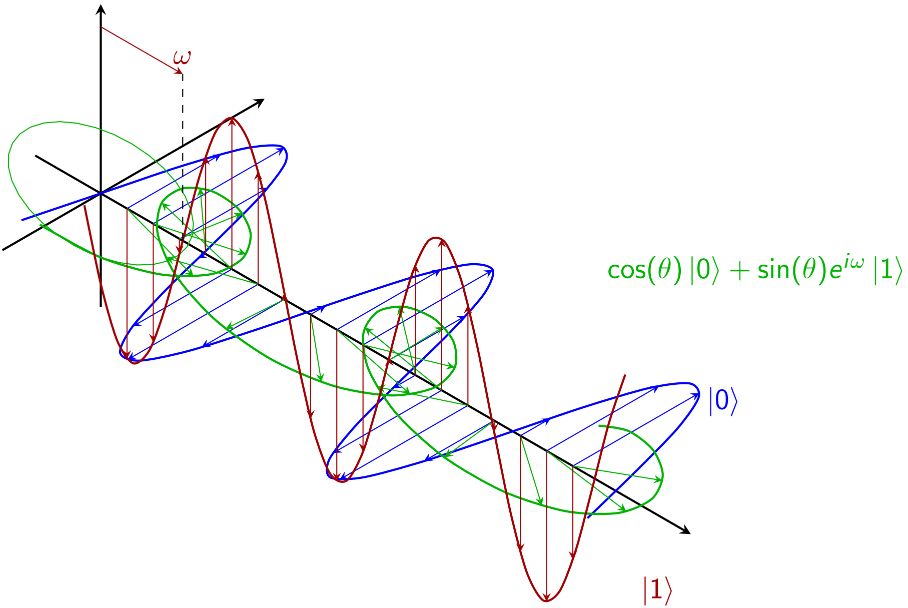
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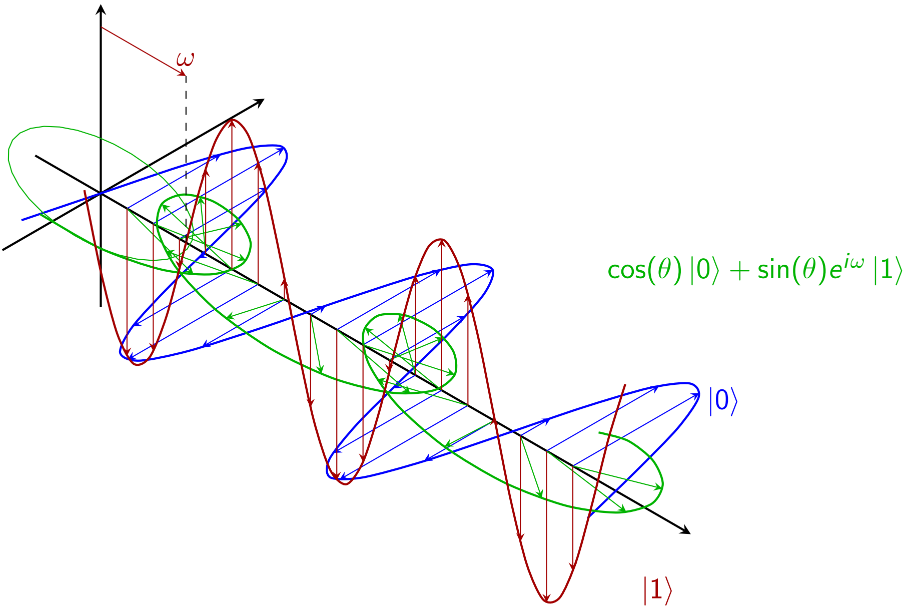

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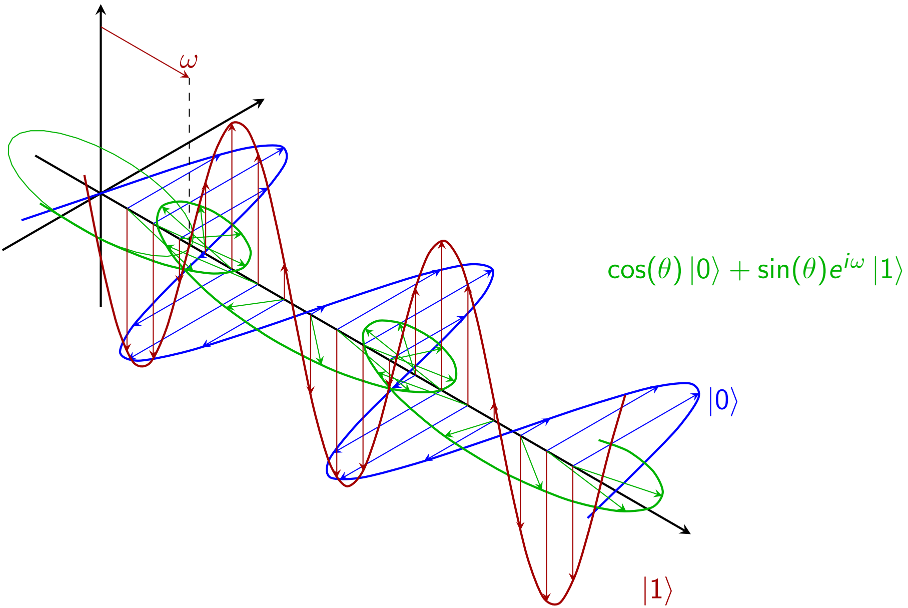
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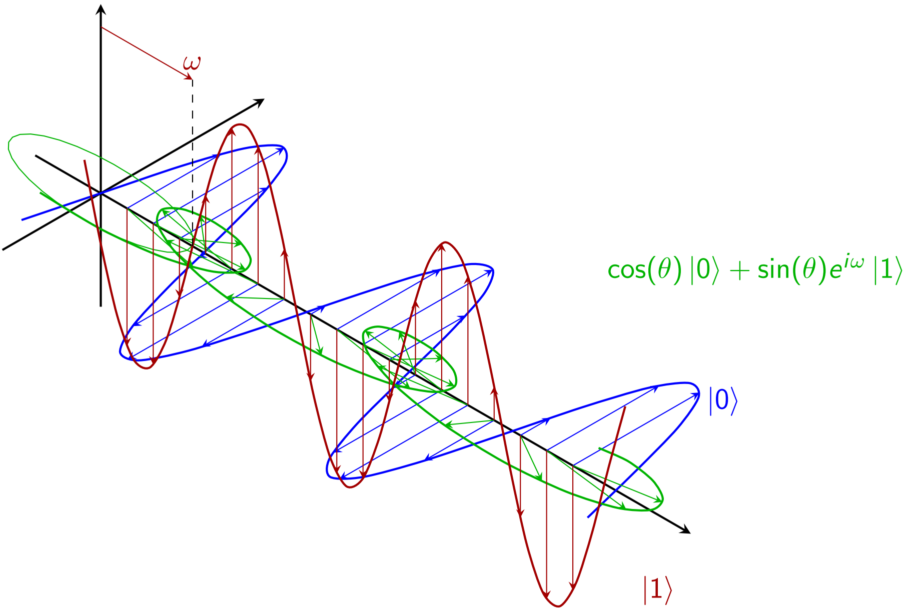
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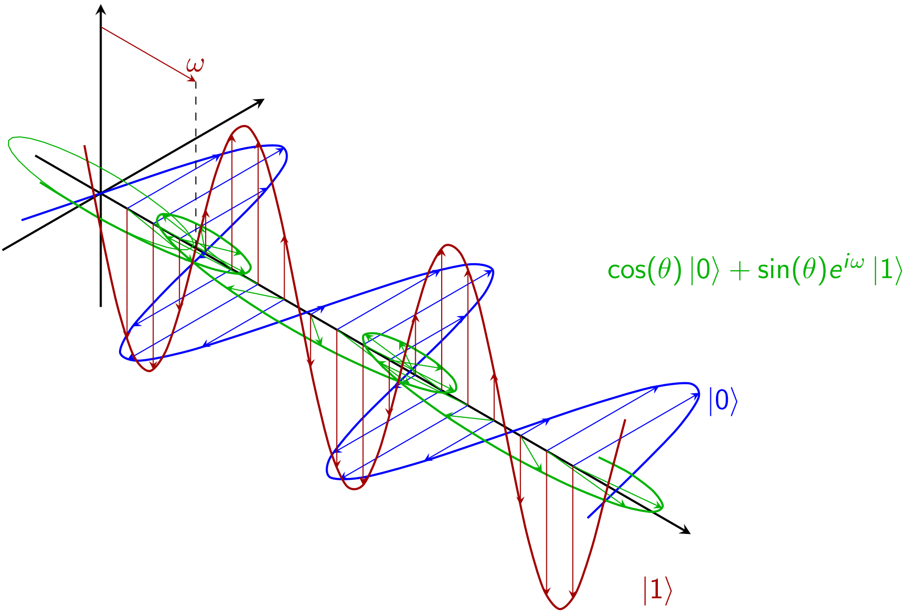
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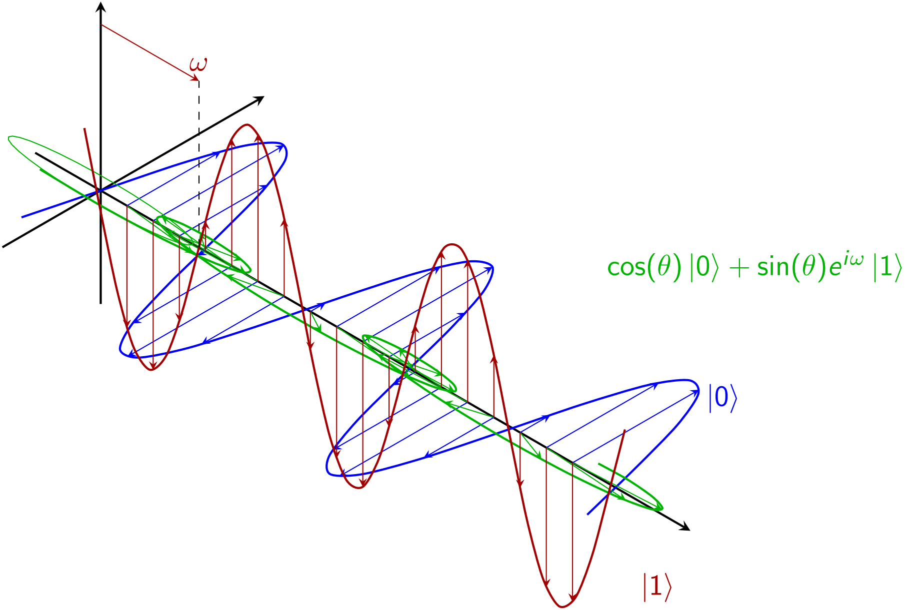
$|1\rangle$



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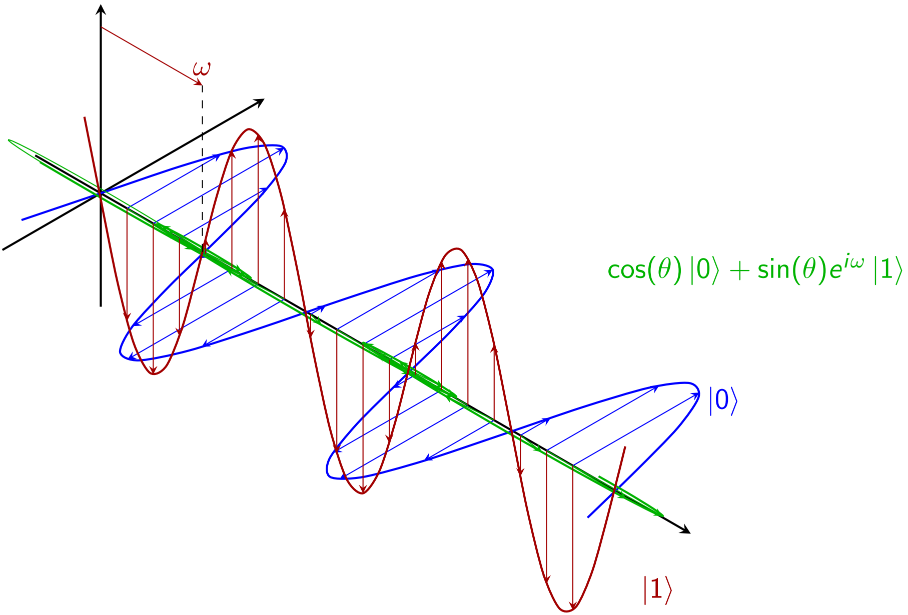

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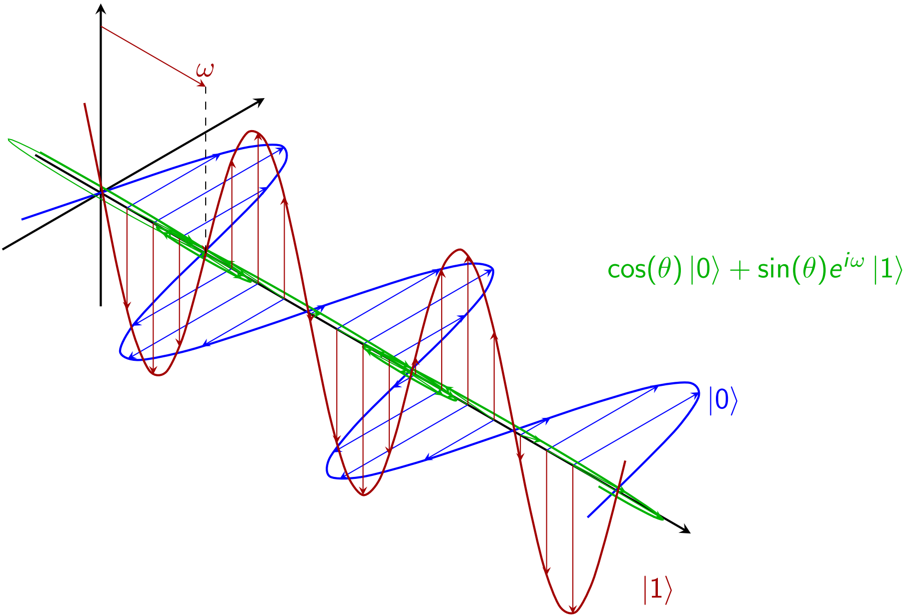

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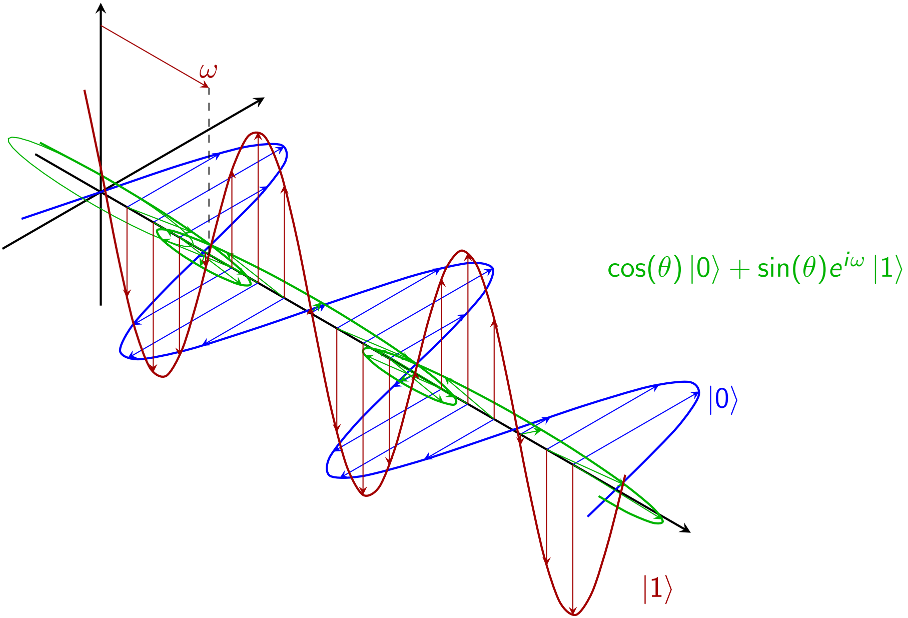
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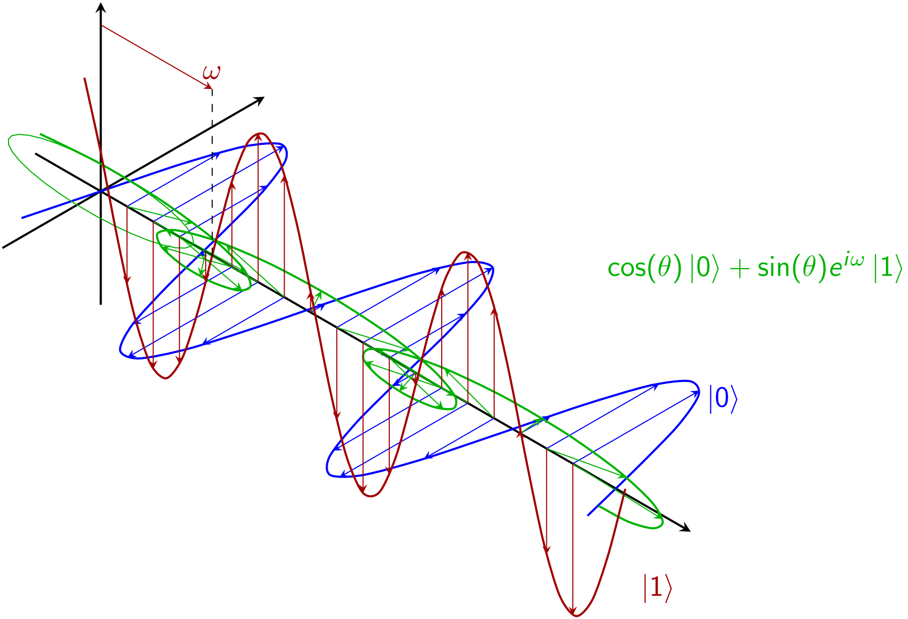

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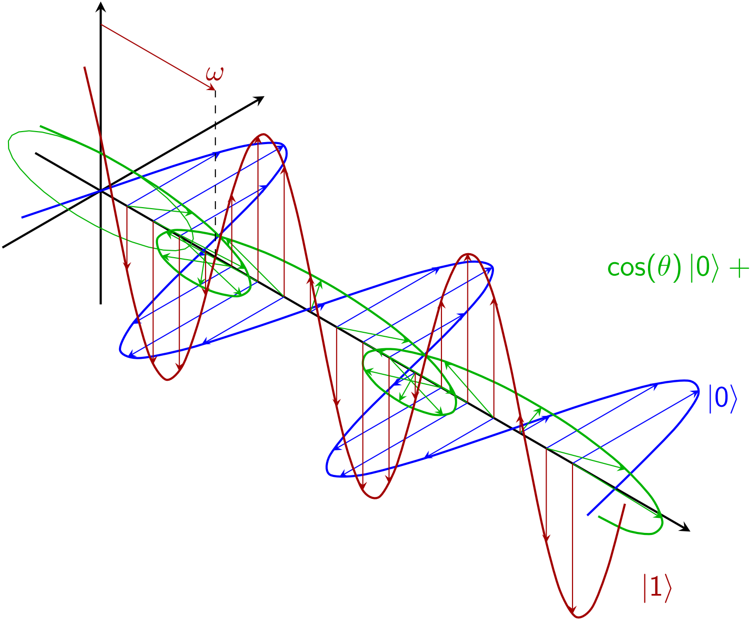

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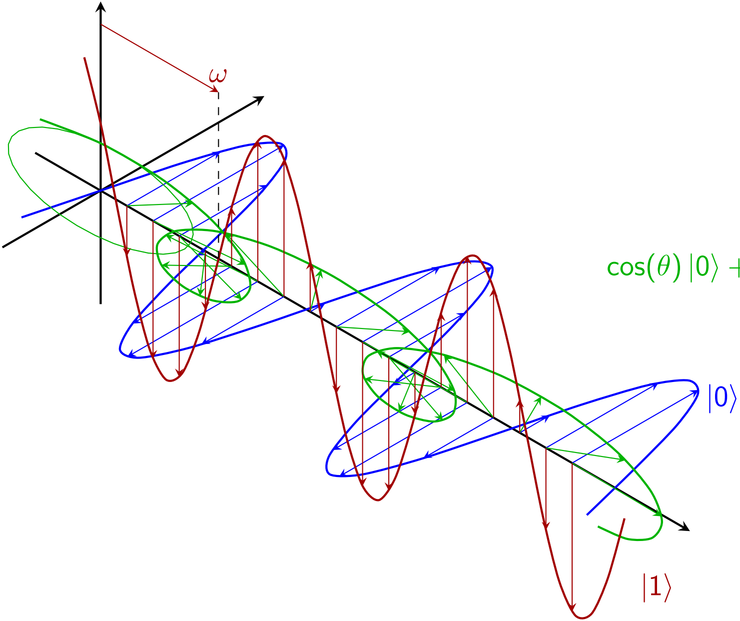

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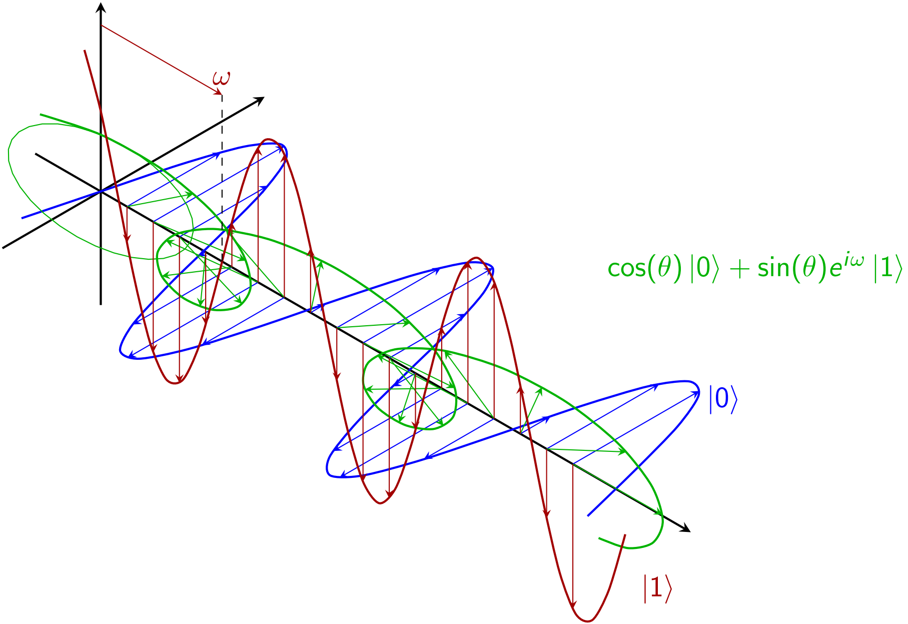

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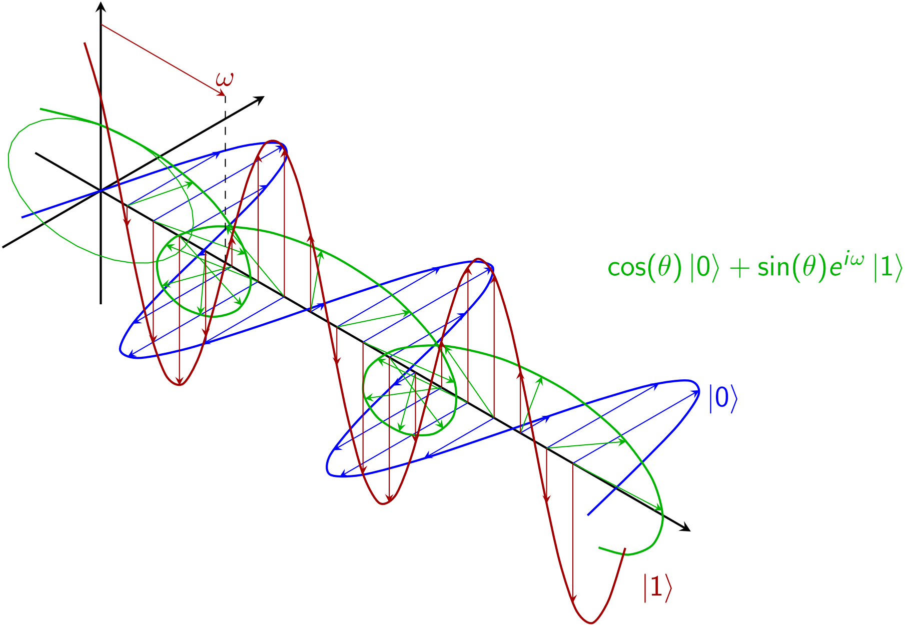
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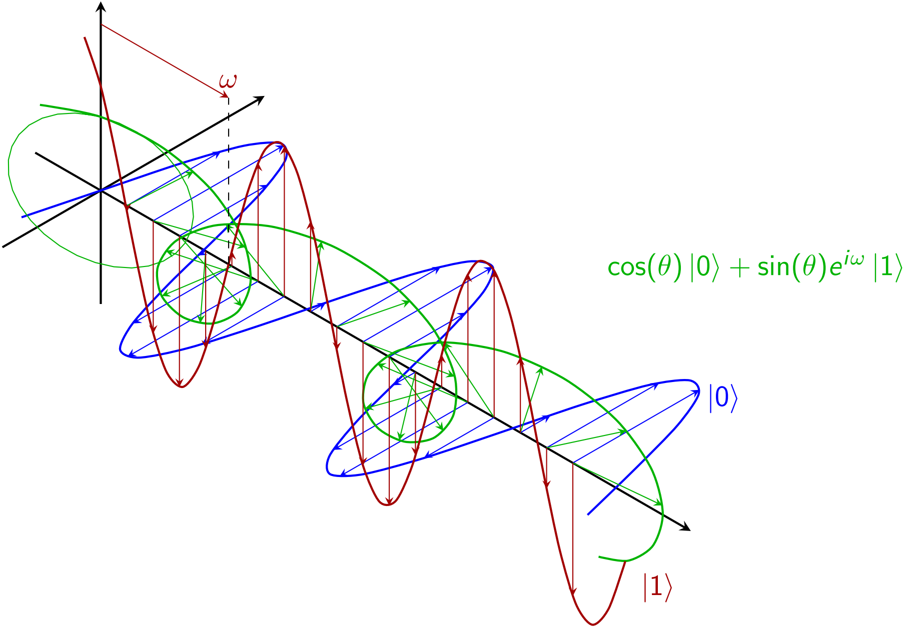
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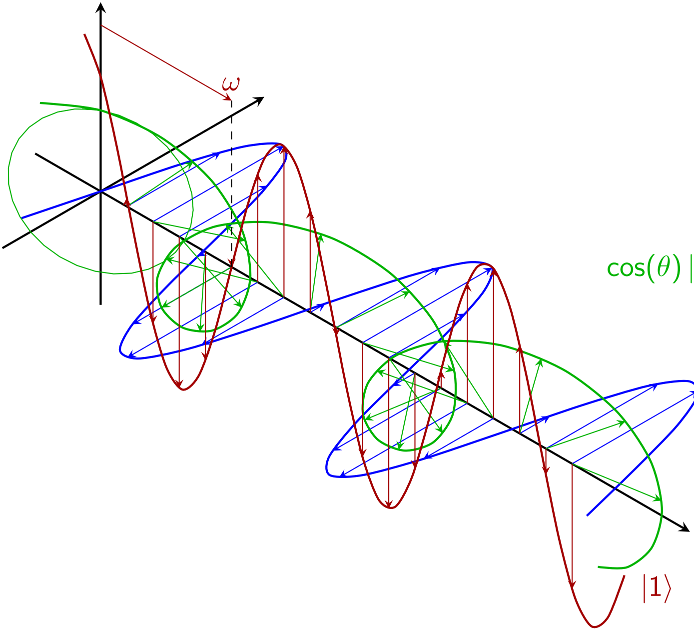

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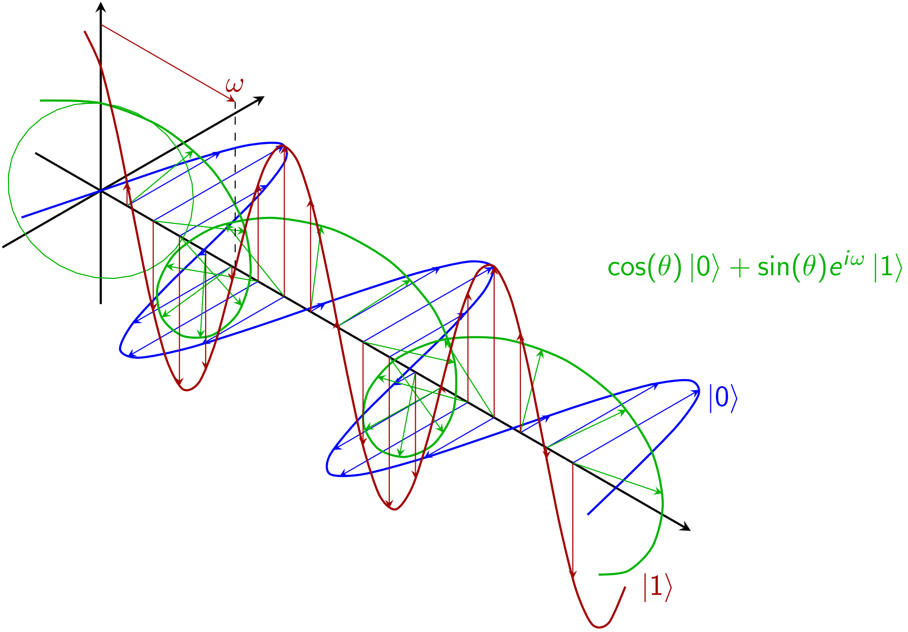

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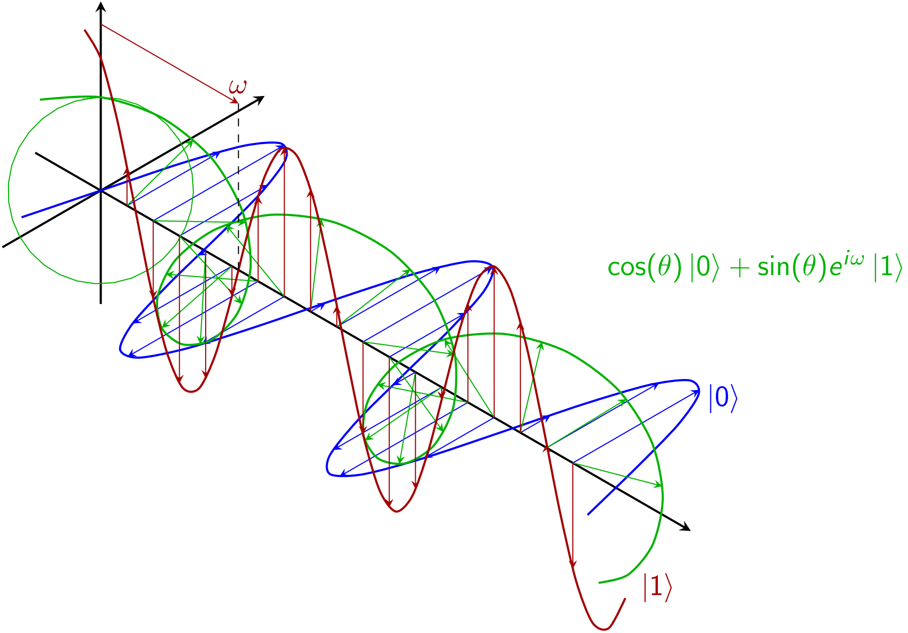

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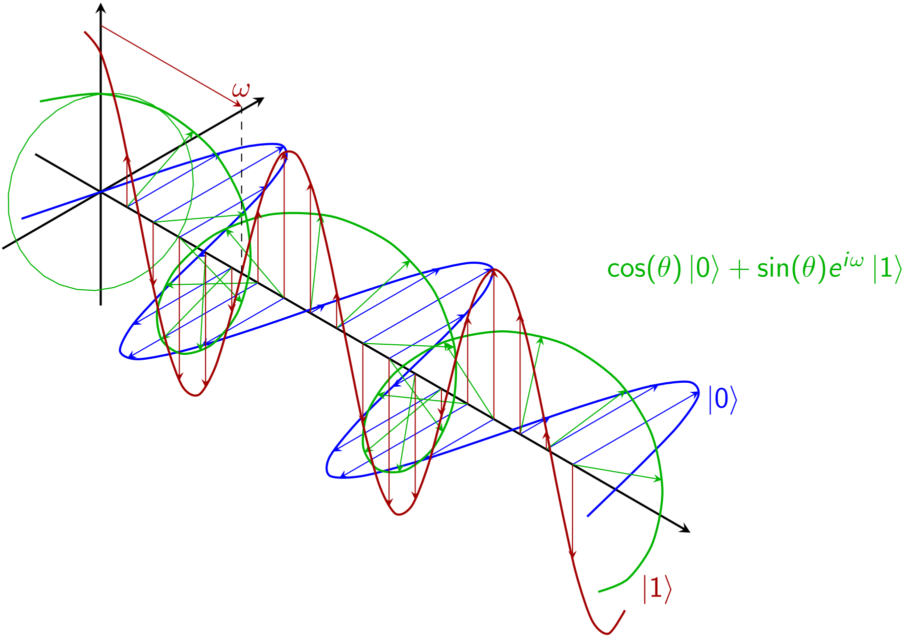
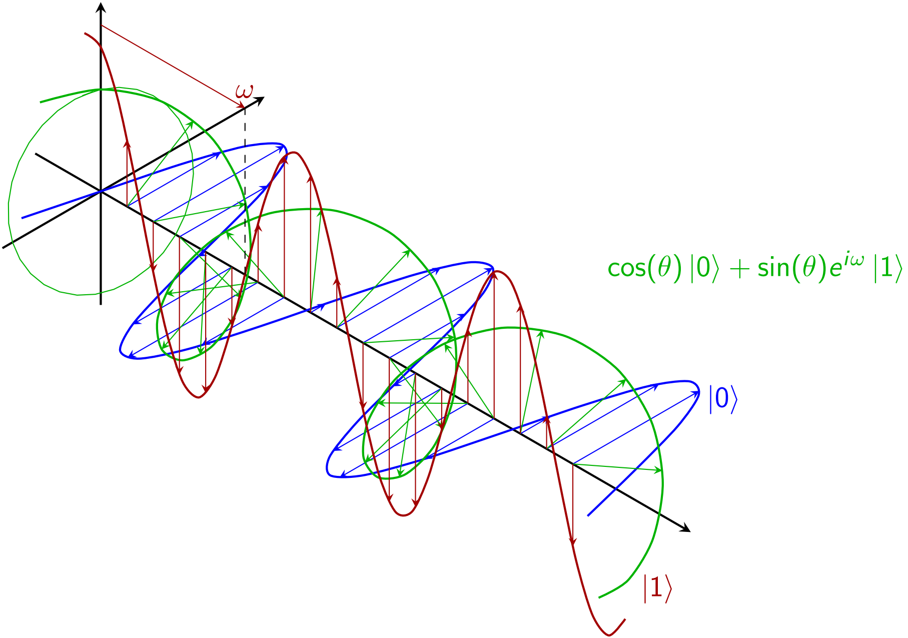

ω
cos(θ) |0⟩ + sin(θ)e^{iω} |1⟩
|0⟩
|1⟩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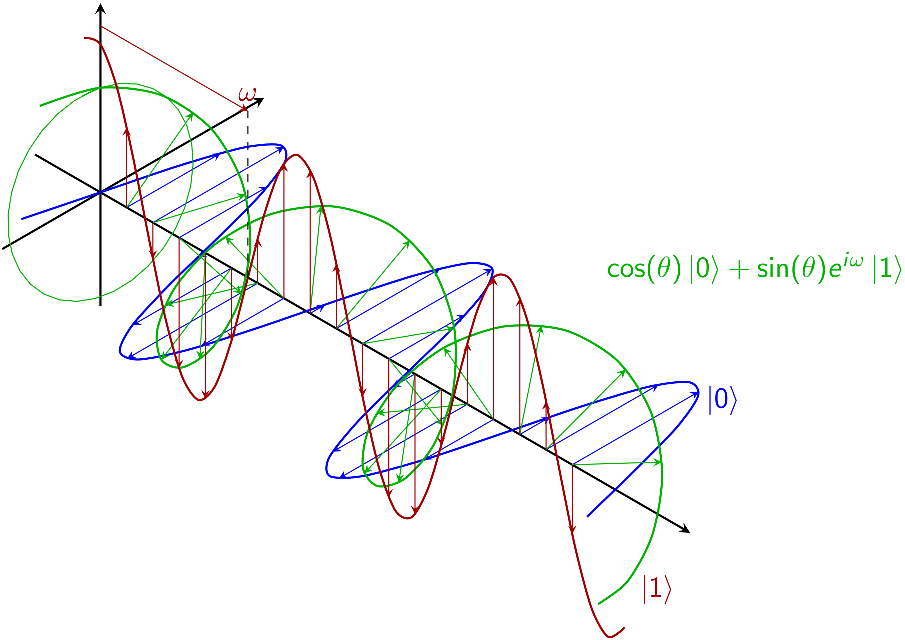

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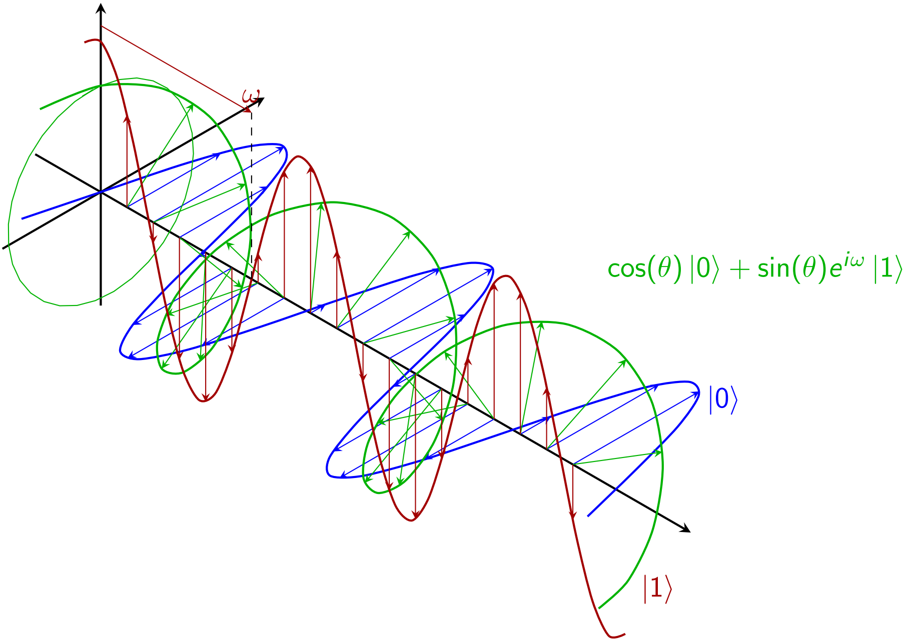

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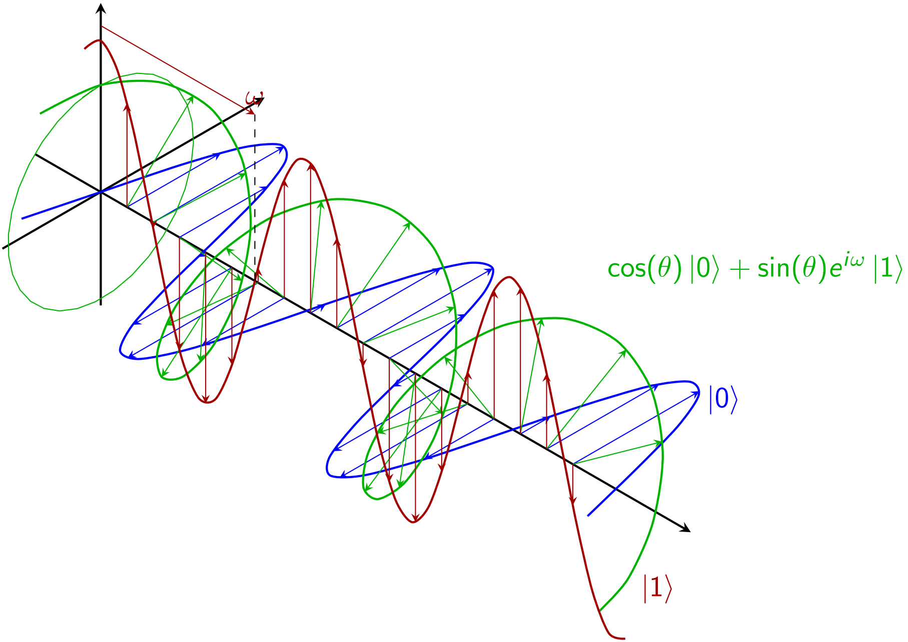

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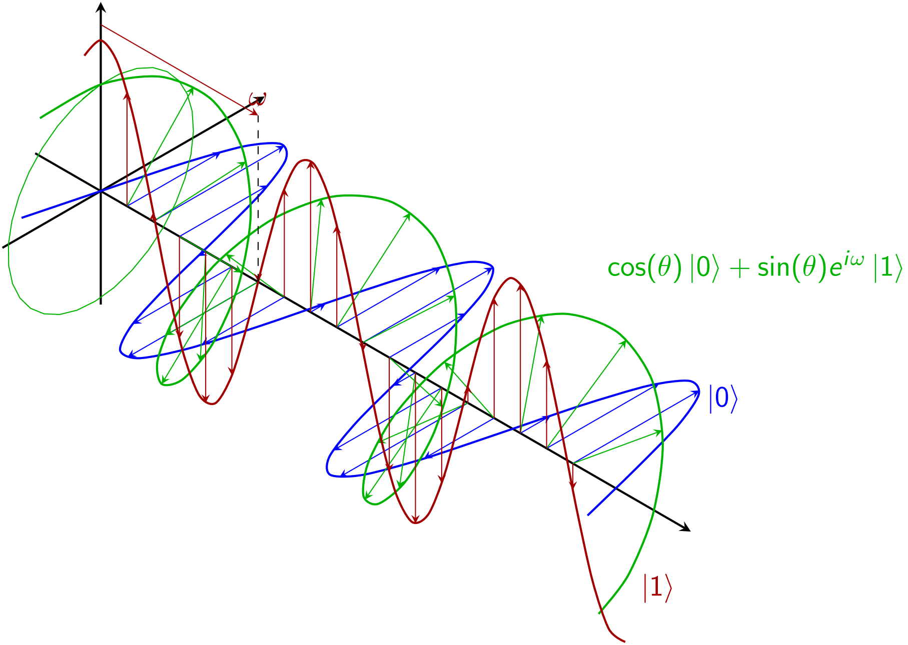

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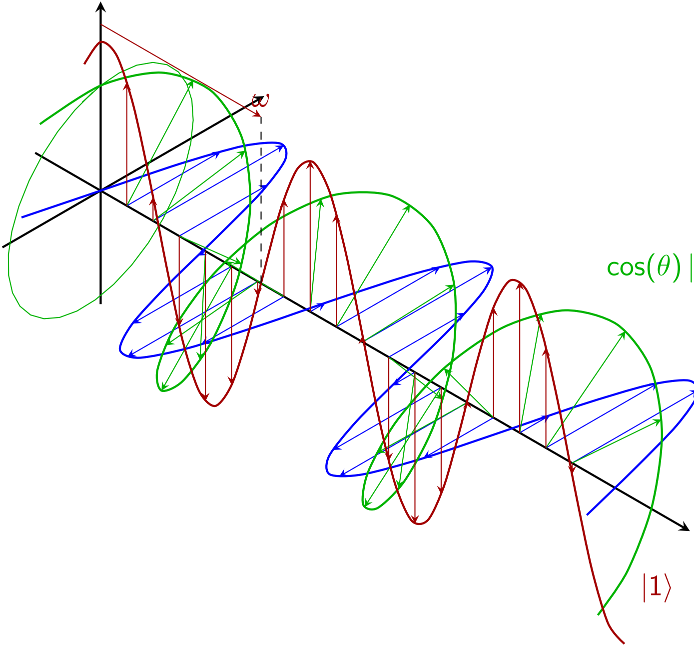

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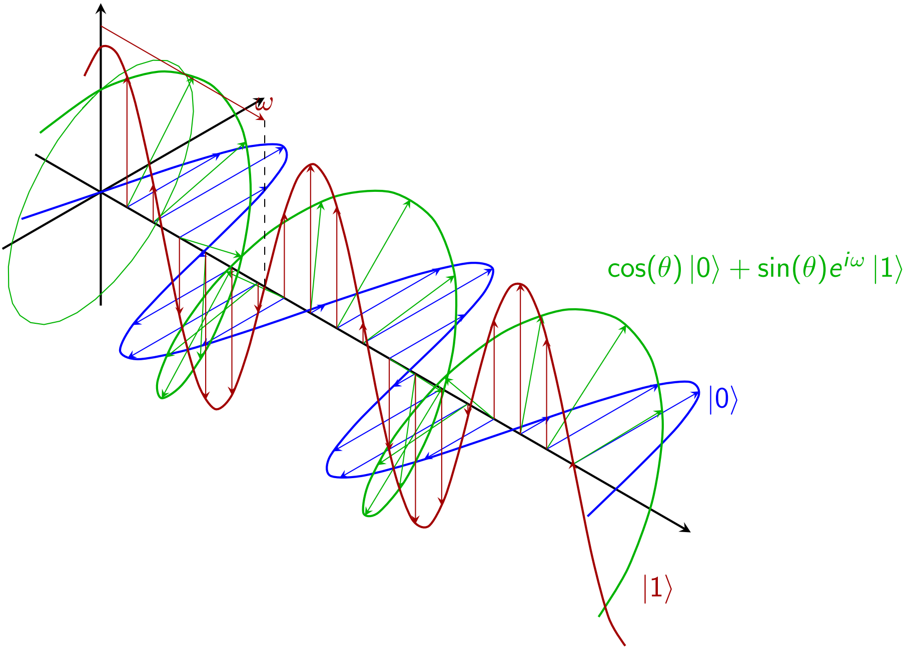
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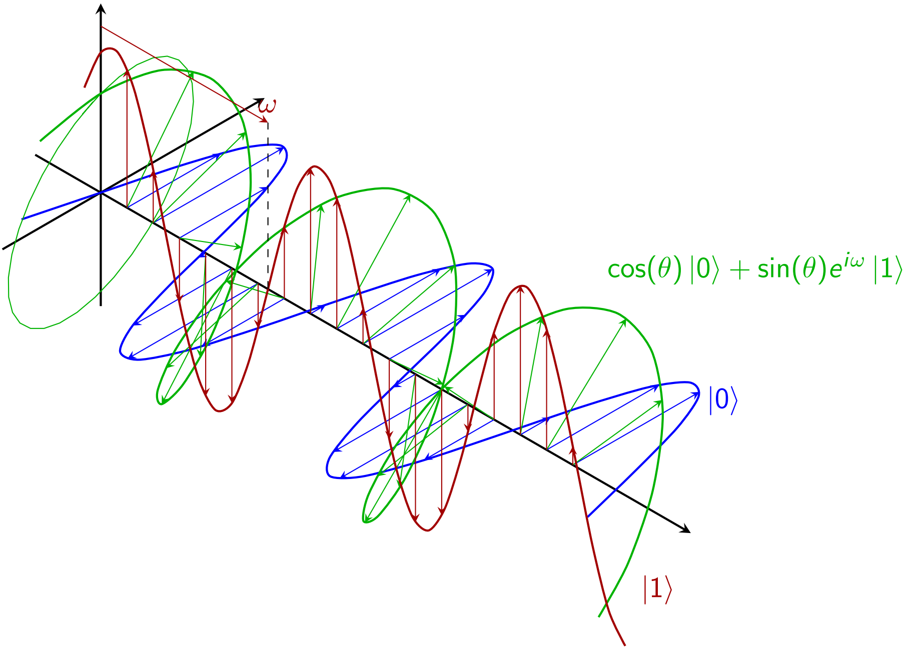
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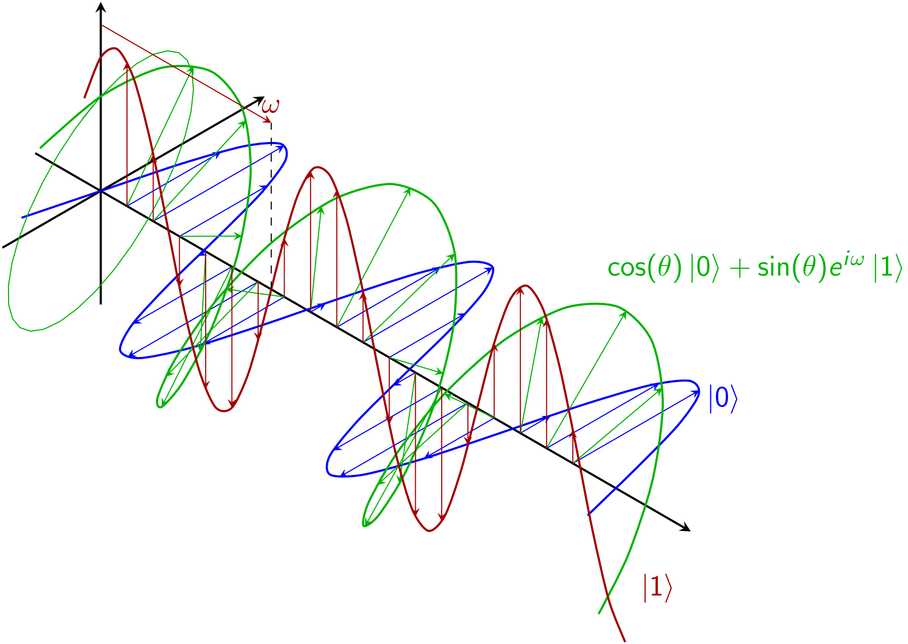

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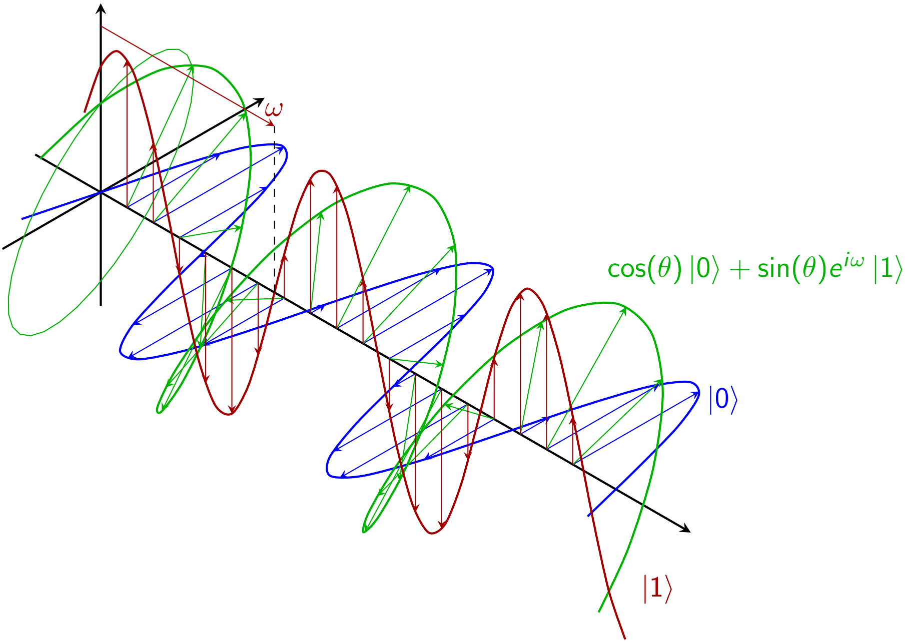
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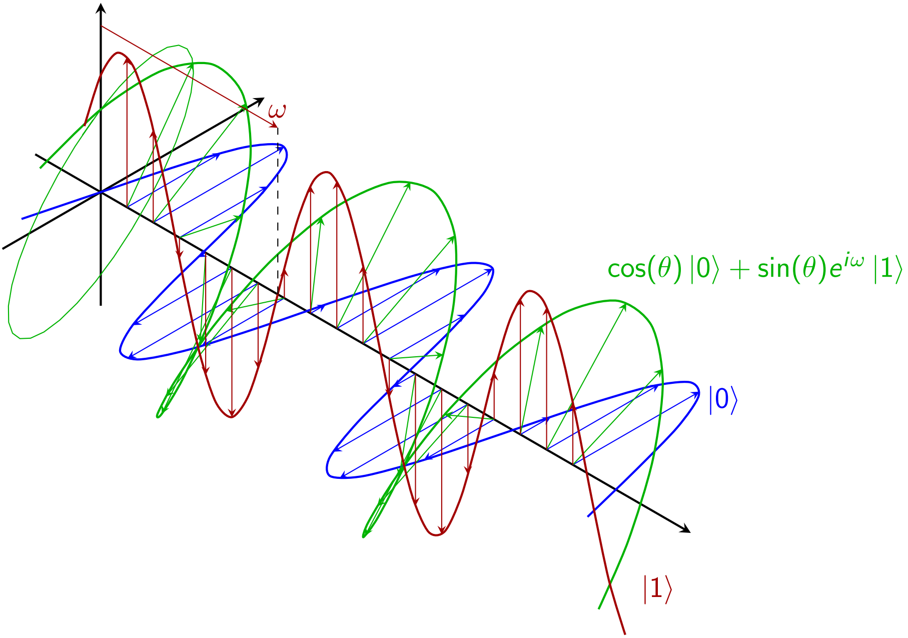
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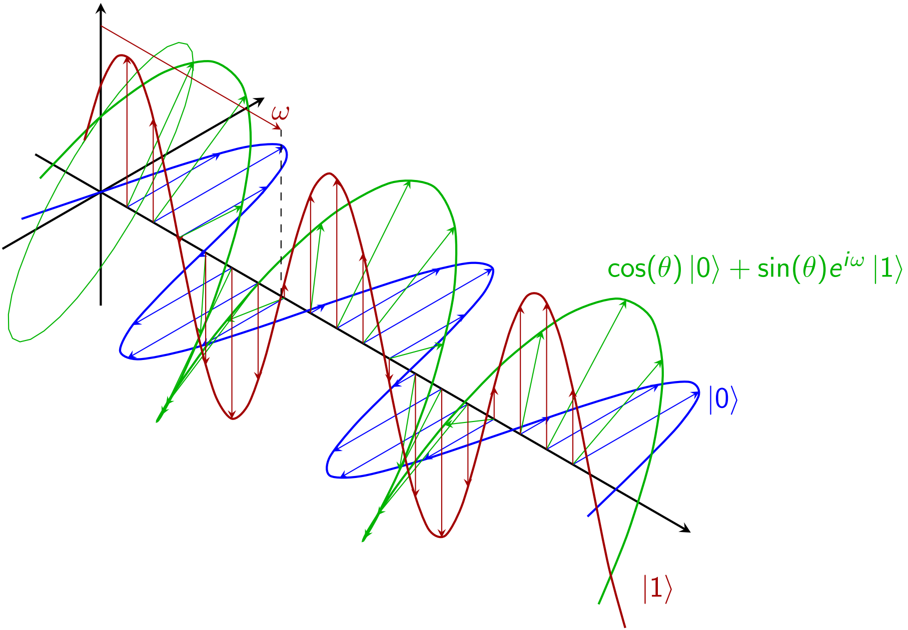

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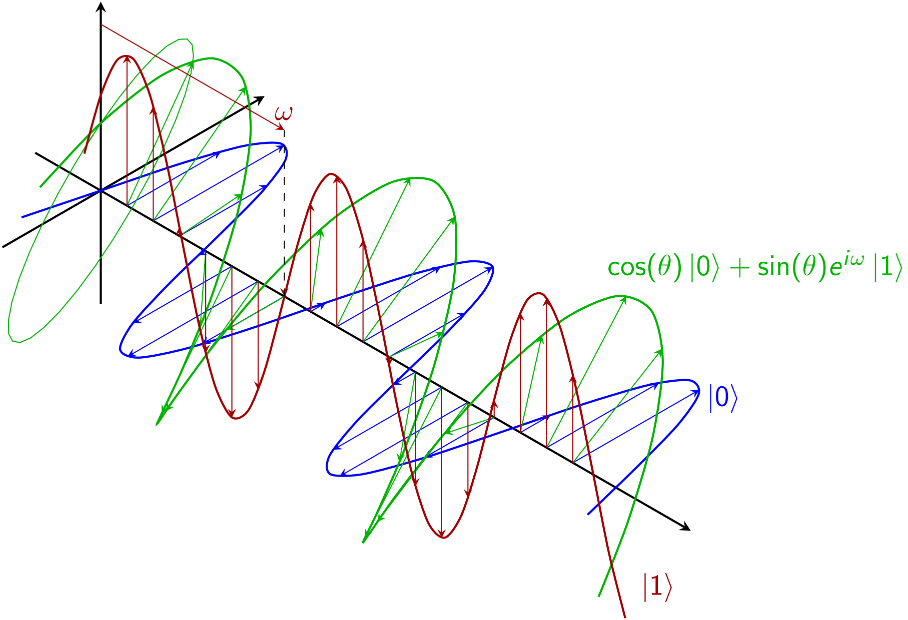
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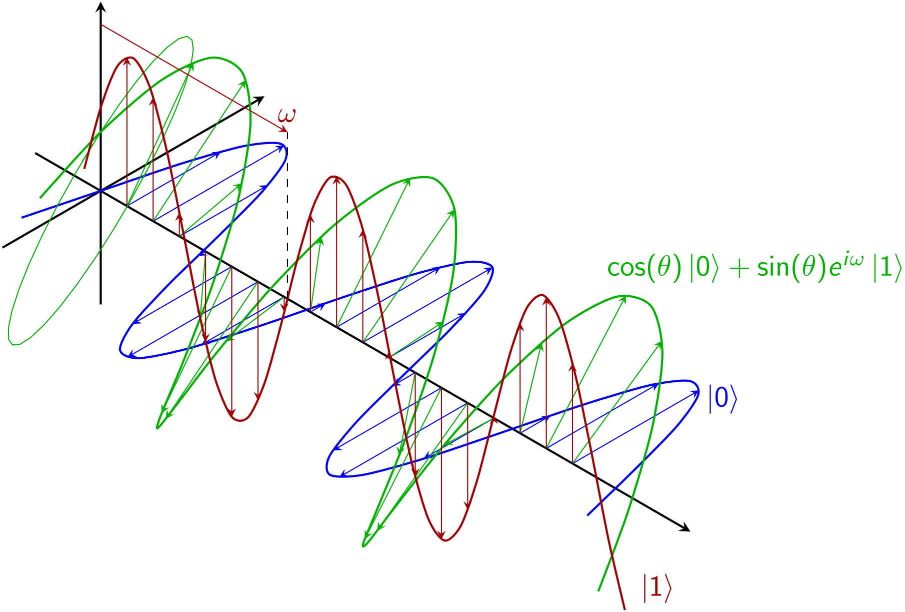

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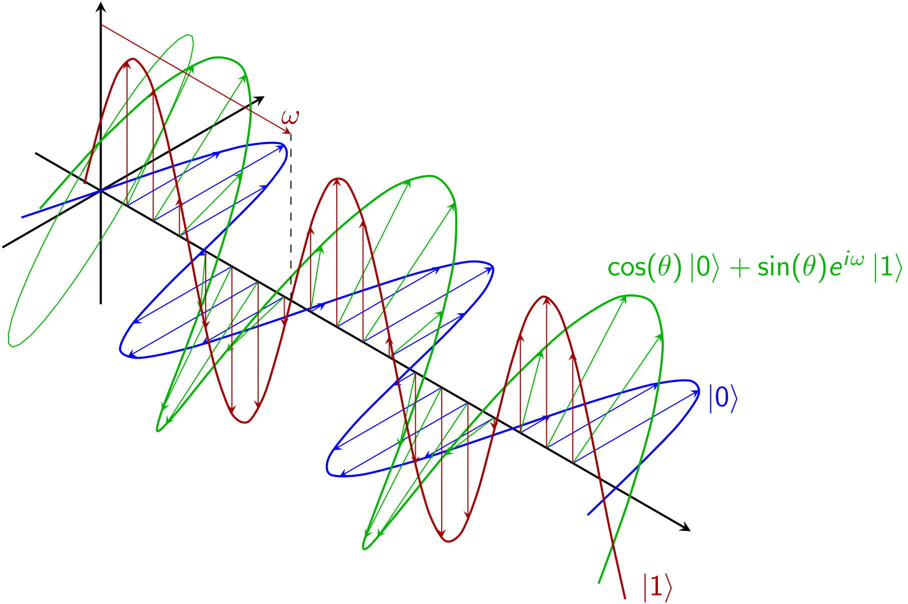
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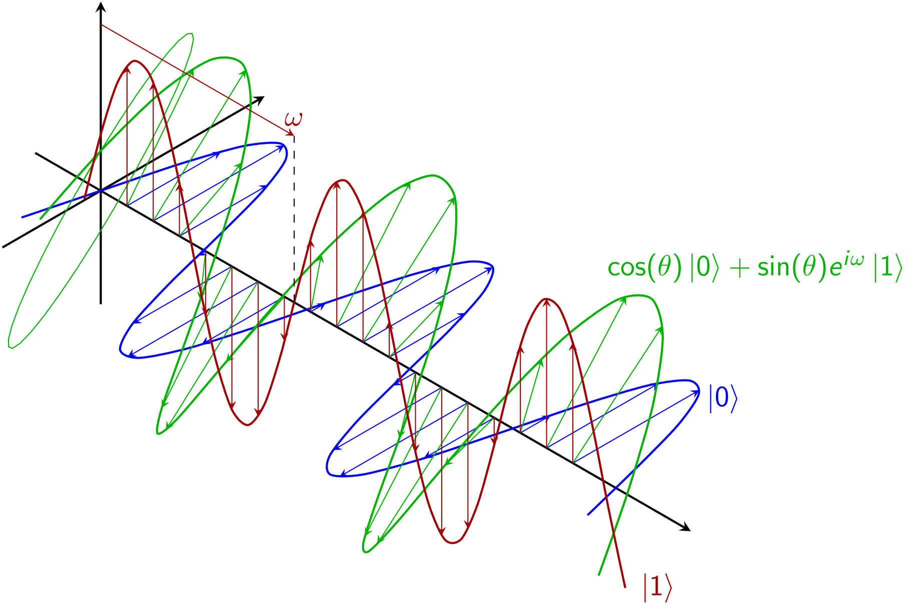
ω
cos(θ) |0⟩ + sin(θ)e^{iω} |1⟩
|0⟩
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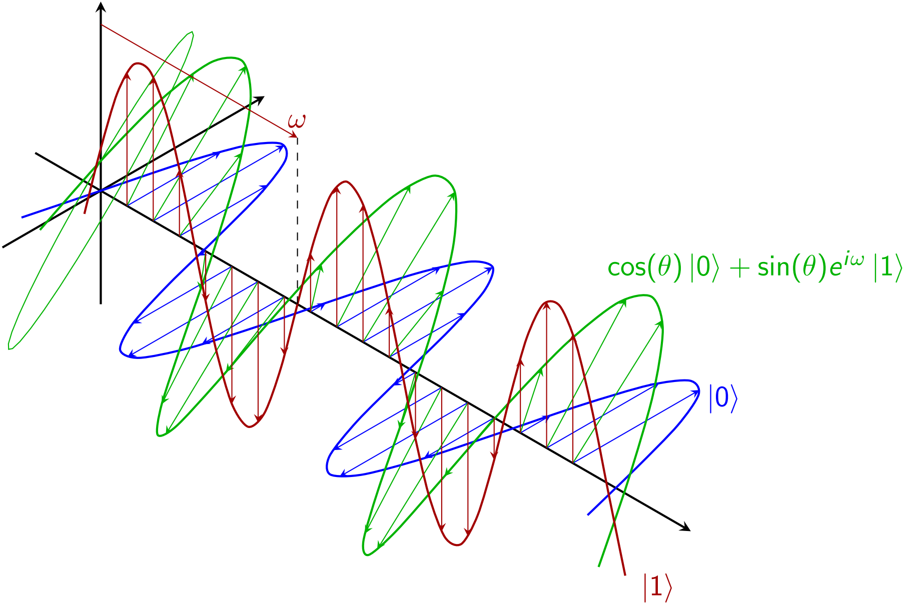

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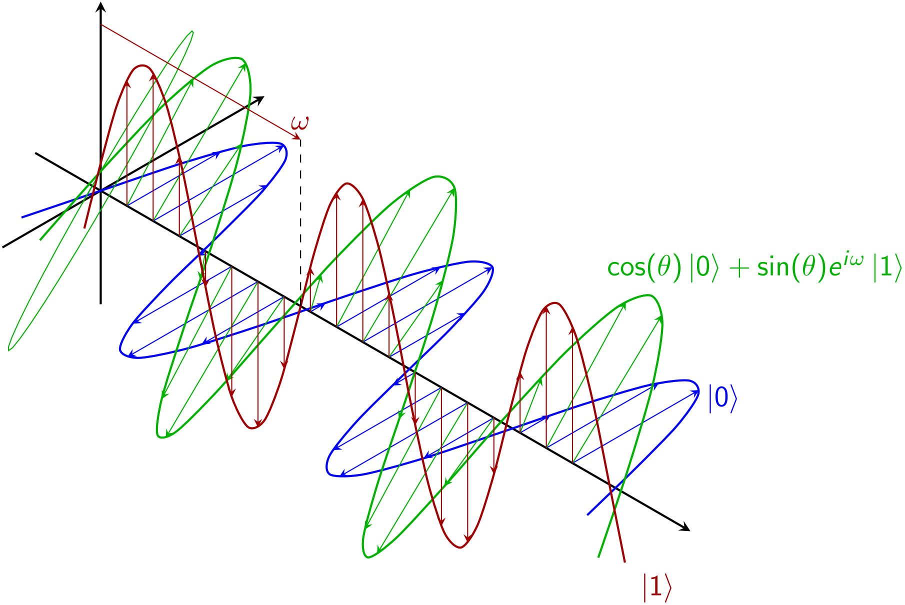
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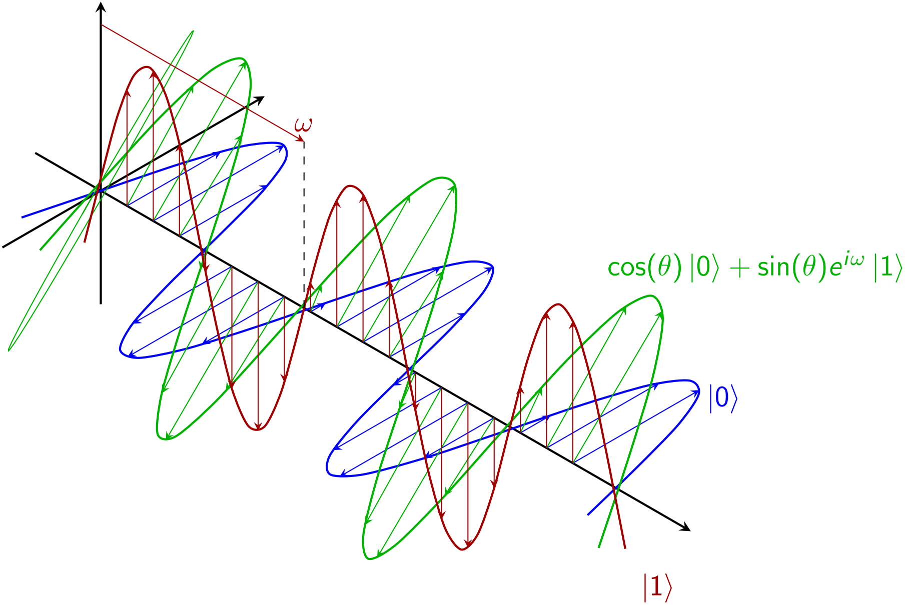
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Return